

Judiciously chosen problems can be the best way of guiding the learner toward enlightenment.

**0.0.2. Structure.** You will quickly notice that everything is numbered by chapter and section, and everything is numbered the same way after that (for ease of reference), except exercises are indicated by letters (and are sprinkled throughout the text, rather than at the end of sections). Individual paragraphs often get numbers for ease of reference, or to indicate a new topic. Definitions are in bold, and are sometimes given in passing.

**0.0.3. Acknowledgments.**

This one is going to be really hard, so I'll write this later. (Mike Stay is the author of Jokes 1.3.11 and 21.5.2.) The phrase “The Rising Sea” is due to Grothendieck [Gr6, p. 552-3], with this particular translation by McLarty [Mc, p. 1], and popularized as the title of Daniel Murfet’s excellent blog [Mur].

## 0.1 For the reader

*This is your last chance. After this, there is no turning back. You take the blue pill, the story ends, you wake up in your bed and believe whatever you want to believe. You take the red pill, you stay in Wonderland and I show you how deep the rabbit-hole goes.*  
— Morpheus

The contents of this book are intended to be a collection of communal wisdom, necessarily distilled through an imperfect filter. I wish to say a few words on how you might use it, although it is not clear to me if you should or will follow this advice.

Before discussing details, I want to say clearly at the outset: the wonderful machine of modern algebraic geometry was created to understand basic and naive questions about geometry (broadly construed). The purpose of this book is to give you a thorough foundation in these powerful ideas. *Do not be seduced by the lotus-eaters into infatuation with untethered abstraction.* Hold tight to your geometric motivation as you learn the formal structures which have proved to be so effective in studying fundamental questions. When introduced to a new idea, always ask why you should care. Do not expect an answer right away, but demand an answer eventually. Try at least to apply any new abstraction to some concrete example you can understand well.

Understanding algebraic geometry is often thought to be hard because it consists of large complicated pieces of machinery. In fact the opposite is true; to switch metaphors, rather than being narrow and deep, algebraic geometry is shallow but extremely broad. It is built out of a large number of very small parts, in keeping with Grothendieck’s vision of mathematics. It is a challenge to hold the entire organic structure, with its messy interconnections, in your head.

A reasonable place to start is with the idea of “affine complex varieties”: subsets of  $\mathbb{C}^n$  cut out by some polynomial equations. Your geometric intuition can immediately come into play — you may already have some ideas or questions about dimension, or smoothness, or solutions over subfields such as  $\mathbb{R}$  or  $\mathbb{Q}$ . Wiser heads