

# Synchronization in Random Geometric Graphs

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## Introduction

The Kuramoto model is a popular model to describe synchronization of a system of  $n$  coupled oscillators. If the connections between the oscillators are determined by a given graph, then the dynamics of the model can be described as follows:

$$\begin{cases} \dot{u}_i = \frac{1}{n} \sum_{j \sim i} \sin(u_j - u_i) \\ u_i(0) = \bar{u}_i \end{cases}.$$

Global synchronization means that for almost any initial condition, the system converges to a state where all phases coincide.

## Initial question

For a given graph, is it possible to guarantee global synchronization, or can there be patterns?

## Random Geometric Graphs (RGG)

A RGG with parameters  $n$  and  $\epsilon_n$  in a space  $D$  is a graph  $G_n(V, E)$  defined as follows:

- $V = \{x_0, x_1, \dots, x_{n-1}\} \subseteq D$  is a sample of  $n$  independent and uniformly distributed random variables;
- $\{x_i, x_j\} \in E$  if  $d(x_i, x_j) < \epsilon_n$ .

In this framework, the Kuramoto model takes the form

$$\begin{cases} \dot{u}_i = \frac{1}{n^2 \epsilon_n^3} \sum_{j \sim i} \sin(u_j - u_i) \\ u_i(0) = \bar{u}_i \end{cases} \quad \longleftrightarrow \quad \begin{cases} \dot{u} = -\nabla E_n(u) \\ u(0) = \bar{u} \end{cases}$$

where

$$E_n(u) = \frac{\pi}{2n^2 \epsilon_n^3} \sum_{i=1}^n \sum_{j \sim i} (1 - \cos(u_j - u_i)).$$

## Main results

- ◆  $D = \mathbb{S}^1$  : There are patterns.

*Approach:* for each  $q \in \mathbb{Z}$ , the Kuramoto energy functional has a local minima of index  $q$  with high probability.

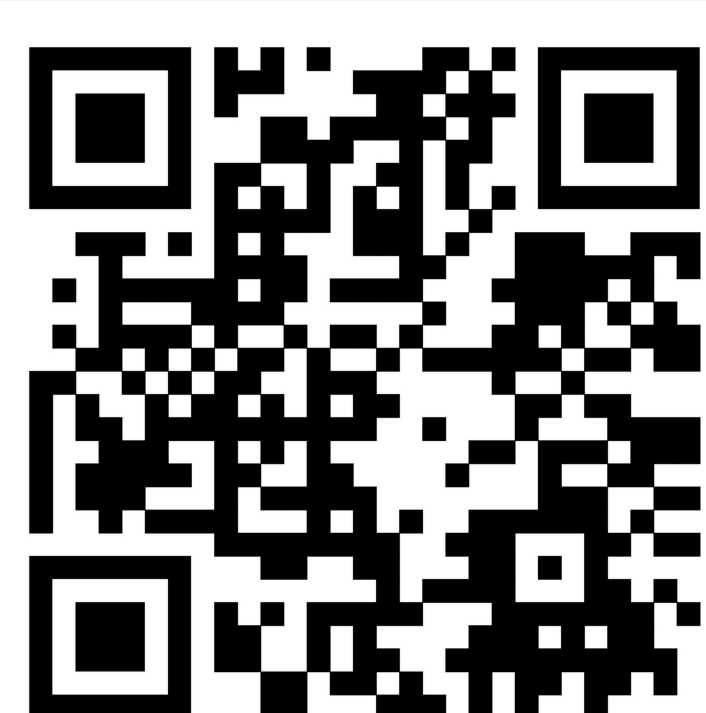
- ◆  $D = \mathbb{S}^2$  : As long as the initial conditions converge to a smooth function, there is global synchronization.

*Approach:* solutions to the Kuramoto model converge to solutions of the heat equation on the sphere with values in  $\mathbb{S}^1$ , if the initial conditions do so.

## Final question

What topological properties of the space  $D$  might determine the possibility of global synchronization?

## QR to the articles



Joint work with  
J. Fernández Bonder  
and P. Groisman



Joint work with  
P. Groisman  
and R. Huang