# Synchronization in Random Geometric Graphs

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## Introduction

The Kuramoto model is a popular model to describe synchronization of a system of $n$ coupled oscillators. If the connections between the oscillators are determined by a given graph, then the dynamics of the model can be described as follows:

$$\begin{cases} \dot{u}_i = \frac{1}{n} \sum\limits_{j \sim i} \sin(u_j - u_i) \\ u_i(0) = \overline{u_i} \end{cases}.$$

Global synchronization means that for almost any initial condition, the system converges to a state where all phases coincide.

## Initial question

For a given graph, is it possible to guarantee global synchronization, or can there be patterns?

## Random Geometric Graphs (RGG)

A RGG with parameters $n$ and $\epsilon_n$ in a space $D$ is a graph $G_n(V, E)$ defined as follows:

- $V = \{x_0, x_1, \ldots, x_{n-1}\} \subseteq D$ is a sample of $n$ independent and uniformly distributed random variables;
- $\{x_i, x_j\} \in E$ if $d(x_i, x_j) < \epsilon_n$.

In this framework, the Kuramoto model takes the form

$$\begin{cases} \dot{u}_i = \frac{1}{n^2 \epsilon_n^3} \sum\limits_{j \sim i} \sin(u_j - u_i) \\ u_i(0) = \overline{u_i} \end{cases} \quad \rightsquigarrow \quad \begin{cases} \dot{u} = -\nabla E_n(u) \\ u(0) = \overline{u} \end{cases}$$

where

$$E_n(u) = \frac{\pi}{2n^2 \epsilon_n^3} \sum_{i=1}^{n} \sum_{j \sim i} (1 - \cos(u_j - u_i)).$$

## Main results

✦ $D = \mathbb{S}^1$ : There are patterns.
*Approach: for each $q \in \mathbb{Z}$, the Kuramoto energy functional has a local minima of index $q$ with high probability.*

✦ $D = \mathbb{S}^2$ : As long as the initial conditions converge to a smooth function, there is global synchronization.
*Approach: solutions to the Kuramoto model converge to solutions of the heat equation on the sphere with values in $\mathbb{S}^1$, if the initial conditions do so.*

## Final question

What topological properties of the space $D$ might determine the possibility of global synchronization?

## QR to the articles

Joint work with J. Fernández Bonder and P. Groisman

Joint work with P. Groisman and R. Huang