Other Examples of Principal Ideal Domains that are not Euclidean Domains

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Abstract. It is a well-known and easily established fact that every Euclidean domain is also a principal ideal domain. However, the converse statement is not true, and this is usually shown by exhibiting as a counterexample the ring of algebraic integers in a certain, very specific quadratic field, and the proof that this works is quite unnatural and technical. I will present a family of counterexamples constructed using real closed fields.

Certificates of Nonnegativity of Polynomials

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Abstract. We study the problem of certifying the nonnegativity of univariate and multivariate polynomials with rational coefficients. We present new certificates based on representing the input polynomial as a sum of squares modulo specially constructed ideals, which guarantees the desired nonnegativity property. In addition, we introduce new algorithms to compute such certificates. These algorithms have lower complexity than previously known methods and are applicable to any polynomial, without assumptions on radicality or the finiteness of critical points. They can also exploit the sparsity structure of the input polynomial. In the univariate case, we uncover a connection between sums of squares and Karlin's theory of T-systems. This poster is based on joint work with Philipp di Dio, Khazhgali Kozhasov, Elias Tsigaridas, and Chaoping Zhu.

Mollified Christoffel-Darboux kernels and the recovery of densities from moments

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Abstract. A basic problem in optimization and statistics is the recovery of a measure from a given collection of moments. The Christoffel function (and its reproducing Christoffel–Darboux kernel) is a classical tool from approximation theory and orthogonal polynomials, which can be used to approach this task. The Christoffel function allows us to obtain an approximation of the density of the measure as an explicit rational function involving the inverse of the given truncated moment matrix M_n.

Building on recent work by Lasserre, we introduce a novel family of mollified Christoffel functions to improve their quality as density estimators. Our formulation provides an explicit sum-of-squares

(SOS) representation of the reciprocal Christoffel function, which facilitates both efficient computation and theoretical analysis. We also prove quantitative error bounds for density recovery which depends on the regularity of the density and the mollifier. We work for probability measures with compact support in $\mathbb{R}^n$ or on the sphere S^n.

Finally we explore applications of this approach to the estimation of occupation and exit measures for diffusions and also to estimate quasi-stationary distributions for certain stochastic processes using data obtained exclusively from simulations.

Random point configurations on the sphere and logarithmic energy

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Abstract. The logarithmic energy of a configuration of points on the sphere measures its degree of repulsion. In other words, the more separated the points are from each other, the lower the energy and the better the distribution of the configuration. Such repulsive properties are highly desirable in problems of polynomial interpolation on the sphere and, more recently, in machine learning on non-Euclidean geometries.

In this work, we study the average logarithmic energy of the solutions to the polynomial eigenvalue problem for Gaussian random matrices, recovering known results for the Shub-Smale polynomials and the Spherical Ensemble.

Our results suggest that the solutions to this problem behave favorably for spherical interpolation, and therefore could be useful for function learning. This opens up novel approaches to function approximation and could thus play a significant role in geometric machine learning models.

We will present the main result, discuss its implications, and outline possible directions for future research.

(Joint work with Diego Armentano and Marcelo Fiori).

Robustness in biological multivalued networks

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Abstract. Absolute concentration robustness (ACR) is an important property of some biochemical networks for which the value of a given species is the same in any positive steady state, regardless of the initial condition. This concept was first formally defined and analyzed in [7], and has since been further studied in several articles [3, 4, 5, 6]. We consider the framework of *Multivalued Logic* [1] introduced in [2], where the concentrations of the network species $x_1, \ldots, x_n$ take values in a finite set $X_m = \{0, \frac{1}{m}, \frac{2}{m}, \ldots, 1\}$ of arbitrary size $m+1$. The operations used are $\mathrm{neg}(x) = 1 - x$, $x \oplus y = \min\{1, x + y\}$, $x \odot y = \max\{0, x + y - 1\}$, which are well suited to biological interpretation. In these discrete-time dynamical systems, the network dynamics is described by $F : X_m^n \to X_m^n$, whose iterations determine the evolution of the system, and steady states correspond to vectors $\mathbf{x} \in X_m^n$ satisfying $F(\mathbf{x}) = \mathbf{x}$.

We introduce the notion of ACR in multivalued networks, present theoretical results to rule out the presence of ACR or characterize networks with certain type of ACR, and analyze specific cases. Some examples were addressed computationally, while others, biologically significant, were studied from a theoretical perspective. This is joint work with M. Pérez Millán.

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Códigos LRC con tasa de transmisión $R \approx \frac{1}{2}$ sobre torres de cuerpos de funciones

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Abstract. En este trabajo mostraremos la construcción de códigos LRC dada en un trabajo en conjunto con María Chara y Edgar Martinez-Moro, utilizando torres de cuerpos de funciones. Se presentará una construcción general que nos permite obtener códigos LRC lineales de gran longitud $n \approx q^4$, dimensión k y distancia mínima d del orden de q^4, con localidad $r = q - 1$. En particular, comentaremos un ejemplo donde la tasa $R = k/n$ es estrictamente mayor a $1/2$.

A Conway and Coxeter theorem. From mathematical enternainment to hyperbolic triangulations

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Abstract. In the 1970s, two prominent mathematicians, J. H. Conway and H. S. M. Coxeter, published a mathematical outreach article challenging the readers to find solutions for some elementary problems. The article, published in two parts—problems and solutions—was titled "Triangulated polygons and frieze patterns." A frieze is an arrangement of positive integers that satisfy a local equation of the form $AB - CD = 1$.

From the 2000s onward, there was a resurgence of interest in these results in relation to the cluster algebras of Fomin and Zelevinsky. In this poster, we trace this history, arriving at a generalization of the initial results by interpreting the integers as geodesic lengths on hyperbolic surfaces.

Multistationarity in enzymatic cascades

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Abstract. Steady states of biological networks under mass-action kinetics are the common zeros of a set of polynomial equations. In this work, we review different conditions for the phosphorylation-dephosphorylation enzyme cascade with 3 levels to admit multiple steady states in the same stoichiometric compatibility class. The simplest version of this biological network gives rise to a system of 14 polynomial differential equations with 18 parameters to analyze. In order to solve this complex problem, we explore three algebraic methods to understand the network structure and find parameter spaces that guarantee two or more common solutions. This work is part of my *tesis de licenciatura* with Alicia Dickenstein.

Extendability of foliations

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Abstract. Given a foliation $\mathcal{F}$ on X and an embedding $X \subseteq Y$, is there a foliation on Y extending $\mathcal{F}$? Using formal methods, we show that this question has an affirmative answer whenever the embedding is sufficiently positive with respect to $(X, \mathcal{F})$ and the singularities of $\mathcal{F}$ belong to a certain class. These tools also apply in the case where Y is the total space of a deformation of X. Regarding the uniqueness of the extension, we prove a foliated version of a statement by Fujita and Grauert ensuring the existence of tubular neighborhoods. We also give sufficient conditions for a foliation to have only trivial unfoldings, generalizing a result due to Gómez-Mont. This is a joint work with Sebastián Velazquez.

Optimal Expander Graphs for Sensing Matrix Design

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Abstract. We study the design of sparse measurement matrices for compressive sensing based on locally optimal bipartite expander graphs.

Recovering the sparsest solution of a linear (rectangular) system $Ax = b$ is, in general, NP-hard, but suitably constructed sparse matrices admit extremely efficient recovery algorithms. Expander-based matrices in particular enable linear-time reconstruction via peeling procedures analogous to those used for LDPC decoding.

Our contribution is to exploit **local optimal expansion**, i.e., achieving the best possible expansion constants for small subsets of vertices. We provide a characterization of such graphs in terms of their shortest cycles (girth), and we develop probabilistic constructions in finite projective spaces that realize these optimal local properties. Numerical experiments show that the resulting matrices achieve substantially higher recovery rates than random sparse matrices with comparable density, and they outperform similar geometric designs that do not enforce optimal local expansion.

This work is being done in collaboration with Marcelo Fiori (IMERL - UDELAR) and Mauricio Velasco (CMAT - UDELAR).

Characterization of Logarithmic Fekete Critical Configurations of at Most Six Points in All Dimensions

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Abstract. We consider the logarithmic Fekete problem, which consists of placing a fixed number of points on the unit sphere in $\mathbb{R}^d$, in such a way that the product of all pairs of mutual Euclidean distances is maximized or, equivalently, so that their logarithmic energy is minimized. Using tools from Computational Algebraic Geometry, we find and classify all critical configurations for this problem when considering at most six points in every dimension d. In particular, our approach gives new proofs of several key results appearing in the literature, with the benefit of using a unified approach

Joint work with Diego Armentano, Leandro Bentancur, Federico Carrasco, Marcelo Fiori and Mauricio Velasco. To appear in the Proceedings of the 2025 International Symposium on Symbolic and Algebraic Computation (ISSAC). Preprint: https://arxiv.org/abs/2502.10152.