MateMax 2025: Titles and abstracts of talks

Data, Geometry and Homology: subjective view on TDA with illustrations from life sciences

Wojciech Chacholski (wojtek@kth.se)
KTH, Stockholm

Abstract. Successful data analysis relies on representing data through objects that are amenable to statistical methods. In recent years, there has been an explosion of applications where homological representations have played a significant role. In this talk, I will introduce one such representation called stable rank, and present several novel approaches for using it to encode geometric information and analyze data. I will provide several illustrative examples of how to use stable ranks to find meaningful results in biological data.

Data Loci in Algebraic and Metric Geometry

Sandra Di Rocco (dirocco@kth.se)
KTH, Stockholm

Abstract. Data loci arise naturally whenever algebraic models exhibit special behavior at specific parameter values (tangency, singularity, or other forms of degeneracy). In the first part of the talk, I will discuss data loci in a projective algebraic setting, where classical constructions such as dual varieties, and discriminants capture when hyperplane sections or systems of equations become singular, including joint work with Alicia.
The focus will then turn to the metric perspective. I will report on recent joint work with Gustafsson and Sodomaco introducing the conditional Euclidean Distance Degree, which counts the critical points of the squared distance to a variety X under the additional requirement that the closest point lies in a subvariety $Z \subset X$. I will also discuss joint work with Rose and Sodomaco on higher-order data loci, highlighting how metric and algebraic viewpoints enrich each other.

Syzygies and sumsets, commutative algebra vs additive combinatorics

Philippe Gimenez (pgimenez@uva.es)
IMUVa - Universidad de Valladolid (Spain)

Abstract. Given a finite nonempty subset $\mathcal{A}$ in $\mathbb{N}^d$, for all $s \geq 0$, the set formed by all sums of s non necessarily different elements in $\mathcal{A}$, $s\mathcal{A} = \{a_1 + \cdots + a_s,\ a_i \in \mathcal{A}\}$, is called the s-fold iterated sumset of $\mathcal{A}$. Additive combinatorics studies sumsets of $\mathcal{A}$ and their cardinality. On the other hand, if one takes an algebraically closed field $\mathbb{K}$ and $\mathcal{A} = \{\mathbf{a}_1, \ldots, \mathbf{a_n}\} \subset \mathbb{N}^d$, one can associate to each $\mathbf{a_i} = (a_{i1}, \ldots, a_{id})$, the monomial $\mathbf{t}^{\mathbf{a}_i} = t_1^{a_{i1}} \times \cdots \times t_d^{a_{id}} \in \mathbb{K}[t_1, \ldots, t_d]$, and define the ring homomorphism $\varphi_{\mathcal{A}}$: $\mathbb{K}[x_1, \ldots, x_n] \to \mathbb{K}[t_1, \ldots, t_d]$, $x_i \mapsto \mathbf{t}^{\mathbf{a}_i}$. This parametrically defines a toric variety and provides a toric ideal $I_{\mathcal{A}} = \ker \varphi_{\mathcal{A}}$. Based on recent results in [1], [2], [3], [4], [5], and some work in progress, we will show in some specific cases (monomial curves, simplicial varieties with at most one singular point) how the sumsets structure of $\mathcal{A}$ is related to the syzygies and, in particular, to the Castelnuovo-Mumford regularity, of the toric ideal $I_{\mathcal{A}}$. This illustrates the interplay between additive combinatorics and commutative algebra, exhibiting how each area can help to solve problems in the other one.

This is based on joint work with Mario González-Sánchez (U. Valladolid) and Ignacio García-Marco (U. La Laguna)

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Synchronization and Non-Synchronization in Random Geometric Graphs

Pablo Groisman (pgroisma@dm.uba.ar)
Universidad de Buenos Aires and IMAS-CONICET

Abstract. Understanding the geometry of the energy function in physical systems with a large number of components is as important as elusive. A similar situation occurs with the loss functions of deep neural networks and other learning procedures. In both cases, any information about the local minima and other critical points is essential, as they govern the long-time behavior of gradient descent mechanisms.

The Kuramoto model is a gradient system of ordinary differential equations whose dynamics reproduce the behavior of an ensemble of coupled oscillators. The coupling is determined by a given graph. The goal is to understand which features of the graph determine the geometry of the energy.

I learnt from Alicia that by means of a change of variables, understanding the geometry of Kuramoto's energy can be stated as an algebraic geometry problem. In fact, I learnt from Alicia everything I know about algebraic geometry (thanks Alicia!). Unfortunately, it is not too much (sorry Alicia!).

In view of this, in this talk we will deal with the geometry of Kuramoto's energy with other tools. The goal is to study this energy in random geometric graphs on a given Riemannian manifold and to understand which properties of the underlying manifold govern the number and nature of the energy's local minima.

Equations of toric vector bundles

Milena Hering (mhering@ed.ac.uk)
University of Edinburgh

Abstract. The projectivisation of a very ample vector bundle $\mathcal{E}$ admits a natural embedding into projective space. We describe defining equations for this embedding in the case that the underlying variety and the vector bundle are toric. The main tool is to embedd $\mathbb{P}(\mathcal{E})$ into the projectivisation of

a direct sum of line bundles $\mathcal{F}$ and to describe the equations in the Cox ring of $\mathbb{P}(\mathcal{F})$, which has been studied first by Cox, Cattani, and Dickenstein. This is joint work with Diane Maclagan and Greg Smith.

Algorithmic equidimensional decomposition in the sparse setting

Gabriela Jeronimo (jeronimo@dm.uba.ar)
Universidad de Buenos Aires & CONICET

Abstract. The study of polynomial systems with specific structure has gained significant attention in the last decades. In particular, systems of *sparse* polynomials (namely, polynomials with nonzero coefficients only at prescribed sets of monomials) have become a central topic in the computer algebra framework.

In this talk we will discuss joint work with María Isabel Herrero and Juan Sabia on sparse polynomial system solving. We will present a symbolic probabilistic algorithm that, given an arbitrary system $f_1, \dots, f_m$ in $\mathbb{Q}[x_1, \dots, x_n]$ of sparse polynomials, characterizes completely all the equidimensional components of the affine variety $V = \{\mathbf{x} \in \mathbb{C}^n \mid f_1(\mathbf{x}) = 0, \dots, f_m(\mathbf{x}) = 0\}$. Based on deformation techniques, the algorithm computes a finite set of representative points for each of the equidimensional components of V. The complexity bounds are polynomial in combinatorial invariants associated to the supports of the polynomials involved.

Finiteness of central configurations of n bodies in the plane via tropical geometry

Anton Leykin (leykin@math.gatech.edu)
Georgia Tech

Abstract. We present a new method to attempt proving that, for a given n, there are finitely many equivalence classes of planar central configurations in the Newtonian n-body problem for generic masses. Our technique relies on tropical geometry and massive polyhedral computation. We have completed a computer-assisted proof for $n \leq 5$ and made progress toward a proof for $n = 6$, which is the next unsettled case for this version of Smale's sixth problem. (Joint with Anders N. Jensen.)

Nash Blowup Fails to Resolve Singularities in Dimensions Four and Higher

Álvaro Liendo (aliendo@utalca.cl)
University of Talca

Abstract. Resolution of singularities in characteristic zero was established by Hironaka, whose approach relies on a sequence of blowups along carefully chosen centers. Although effective, this method involves a variety of non-canonical choices. In order to obtain a canonical procedure, Nash proposed the blowup that now bears his name, raising the question of whether its iteration suffices to resolve all singularities. In joint work with Federico Castillo, Daniel Duarte, and Maximiliano Leyton-Álvarez, we construct toric counterexamples showing that in dimensions at least four the iterated Nash blowup does not lead to a resolution. I will discuss these counterexamples and place them in the broader context of the resolution problem.

Normal forms for solving non-linear problems

Bernard Mourrain (bernard.mourrain@inria.fr)
INRIA

Abstract. Non-linear problems are ubiquitous in many domains and applications. They can however be extremely difficult to solve, from a numerical or algorithmic point of view. The search for robust and effective numerical methods to address these challenges related to nonlinearity is still an ongoing topic of research.

Normal form methods provide a uniform way to address these problems. They encompass Groebner basis, border basis, resultant solvers. We will investigate these approaches from a geometric point of view, and describe their correlations with normal forms, via duality. We will illustrate them on problems such as solving polynomial systems, finding optima in semi-algebraic sets, tensor decomposition and geometric modeling problems.

Higher order tangent spaces – the toric case

Ragni Piene (ragnip@math.uio.no)
University of Oslo, Norway

Abstract. Projective algebraic varieties can be described and classified by way of their tangent spaces, leading in particular to polar varieties and dual varieties. A finer study involves the higher order tangent spaces, which give rise to higher order polar and dual varieties, as well as higher order reciprocal polar varieties and distance degrees. For toric varieties this study is of particular interest due to the connection with convex geometry and combinatorics. A particular feature of toric varieties is that their higher order tangent spaces can be interpolated to give a toric variety that osculates the given one. The talk will survey results that are part of, or relevant to, joint work with Alicia.

Agebraic winding numbers

Marie-Françoise Roy (marie-françoise.roy@univ-rennes.fr)
Université de Rennes, France

Abstract. In this paper, we propose a new algebraic winding number and prove that it computes the number of complex roots of a polynomial in a rectangle, including roots on edges or vertices with appropriate counting. The definition makes sense for the algebraic closure C = R[i] of a real closed field R, and the root counting result also holds in this case. We study in detail the properties of the algebraic winding number defined in [1] with respect to complex root counting in rectangles. We extend both winding numbers to rational functions, obtaining then algebraic versions of the argument principle for rectangles.
Joint work with Daniel Perrucci, University of Buenos Aires
[1] M. Eisermann, The fundamental theorem of algebra made effective: an elementary real-algebraic proof via Sturm chains. Amer. Math. Monthly 119 (2012), no. 9, 715–752.

From a different facet

Juan Sabia (sabiajuan@yahoo.com.ar)
Universidad de Buenos Aires - CONICET

Abstract. In this talk, I will comment on some of the outreach and educational work that Alicia carried out, including some projects we worked on together.

The scattering correspondence: particle physics meets algebraic statistics

Hal Schenck (hks0015@auburn.edu)
Auburn University

Abstract. An arrangement of hypersurfaces in projective space is strict normal crossing (SNC) if and only if its Euler discriminant is nonzero. We study the critical loci of arbitrary Laurent monomials in the equations of the smooth hypersurfaces. The family of these loci forms an irreducible variety in the product of two projective spaces, known in algebraic statistics as the likelihood correspondence and in particle physics as the scattering correspondence. We establish an explicit determinantal representation for the minimal generators of the bihomogeneous prime ideal that defines this variety. Joint work with Thomas Kahle, Bernd Sturmfels, and Max Wiesmann.

Toric Geometry of Periodic Operators

Frank Sottile (sottile@tamu.edu)
Texas A&M University

Abstract. The Laplacian of a $\mathbb{Z}^d$-periodic graph, or more generally of a Schrödinger operator, has spectrum (generalized eigenvalues) that is a union of intervals in the real line. More structure is revealed from the vantage of representations of $\mathbb{Z}^d$: The operator becomes a map of free modules over the character ring and the spectrum becomes a real algebraic hypersurface in a torus, with a natural toric compactification. Many natural questions about the spectrum become accessible using the tools of our trade: computational, combinatorial, and real algebraic geometry. I will sketch the background and discuss some results from this perspective.

Maximal Mumford Curves from Planar Graphs

Bernd Sturmfels (bernd@mis.mpg.de)
MPI Leipzig

Abstract. We discuss algebraic curves from the perspectives of real and tropical geometry. A curve of genus g is maximal Mumford (MM) if it has g+1 ovals and g tropical cycles. We construct families of MM curves that are full-dimensional in their moduli space. Our curves deformations of line arrangements, to be illustrated with colorful pictures.

Algebraic tools for recovering measures from moments

Mauricio Velasco (mvelasco@cmat.edu.uy)
Universidad de la República (Uruguay)

Abstract. The moments of a measure μ in a compact set $X \subseteq \mathbb{R}^n$ are the averages of monomials according to μ. A problem with applications ranging from statistical inference to stochastic control is the (possibly approximate) recovery of a measure's density from a collection of moments. In this talk I will discuss several generalizations of the classical Christoffel-Darboux kernel method for carrying out such recovery procedures and prove several quantitative guarantees and showcase some recent applications. The talk will discuss joint work (some ongoing) with several coauthors (L. Bentancur, R.Chhaibi, F. Gamboa, C. Meroni and D. Henrion).

Polynomial Methods for Handwriting Recognition

Stephen M. Watt (smwatt@uwaterloo.ca)
Cheriton School of Computer Science
University of Waterloo, Canada

Abstract. Modern devices such as tablets and telephones capture digital ink strokes as sequences of (x, y, t) points where t is an explicit time coordinate or implicit from a sampling frequency. Most handwriting work has been based on analyzing these point sequences. We take a different approach and treat digital ink strokes as segments of plane curves. This gives a representation for the digital ink that does not depend device resolution or on writing speed. Approximating the curves as polynomials in an orthogonal basis leads to highly efficient recognition explainable in geometric terms.

Framework We consider an ink trace to be a segment of a plane curve $\big(x(s), y(s)\big), s \in [0, L]$. Since we are concerned with handwriting recognition, or other curve classification problems, we want curves that look similar to have similar representations. To achieve this, we choose a geometric parameterization and project the curves onto a low-dimensional function space. For the parameterization, we have found arc length, given by $ds^2 = dx^2 + dy^2$, to be an effective choice. For the low-dimensional function space, choosing an orthogonal polynomial basis allows projection that may approximate the functions $x(s)$ and $y(s)$ as closely as desired. We choose a basis $\{B_i\}$ of $d + 1$ polynomials that are orthogonal with respect to an inner product $\langle\cdot,\cdot\rangle_I$ and obtain the approximation $\big(\hat{x}(s), \hat{y}(s)\big)$ where $\hat{x}(s) = \sum_{i=0}^{d} \hat{x}_i B_i(s) \approx x(s)$ and $\hat{y}(s) = \sum_{i=0}^{d} \hat{y}_i B_i(s) \approx y(s)$. The curves are thus be represented as points in $\mathbb{R}^{2(d+1)}$. The components $\hat{x}_i$ and $\hat{y}_i$ may be obtained by numerical integration, *e.g.* $\hat{x}_i = \langle x, B_i\rangle_I / h_i$ where $h_i = \langle B_i, B_i\rangle_I$ [4].

Curve Similarity The previous approach to determining curve similarity was that of "dynamic time warping", originally proposed in [6]. This is a sequence alignment method where sample points of two curves are matched seeking a sample point correspondence that minimizes the sum of squared distances. Discovering the optimal alignment can be time consuming when the samples are spaced differently along the two curves.

Instead, we take the approach to compute a variational integral between the curves to be compared. If the two curves are $\big(x(s), y(s)\big)$ and $\big(\bar{x}(s), \bar{y}(s)\big)$, then we let $\xi(s) = x(s) - \bar{x}(s)$ and $\eta(s) = y(s) - \bar{y}(s)$ and use the distance measure

$$\|\xi\|_I^2 + \|\eta\|_I^2 = \langle\xi,\xi\rangle_I + \langle\eta,\eta\rangle_I = \Big\langle \sum_{i=0}^{d} \xi_i B_i, \sum_{i=0}^{d} \xi_i B_i \Big\rangle_I + \Big\langle \sum_{i=0}^{d} \eta_i B_i, \sum_{i=0}^{d} \eta_i B_i \Big\rangle_I = \sum_{i=0}^{d} (\xi_i^2 + \eta_i^2) h_i. \quad (1)$$

This reduces the similarity computation from thousands of machine instructions to dozens.

Choice of Basis We may choose any orthogonal basis to obtain the fast comparison of (1), and we have explored the use of Legendre, Chebyshev, Legendre-Sobolev and Chebyshev-Sobolev basis polynomials [1]. The Sobolev variants use inner products of the form $\int_a^b f(s)g(s)w(s)ds + \mu \int_a^b f'(s)g'(s)w(s)ds$ where $w(s)$ is the weight function for the corresponding orthogonal polynomial family. This helps match the the changes of direction of the curves. The Legendre variants have a weight function 1 so the inner products can be integrated in real time, as the curve is being written, and then rescaled in small constant time when the pen is lifted. Since conversion of basis is numerically ill-conditioned, we wish to perform various operations such as differentiation, root finding and gcd in the orthogonal basis representation, so we have developed algorithms to do this for bases of interest.

Consequences of Linear Separability When characters are labelled with their type, we find that their coefficient vectors form clusters, linearly separable by type, with a few only very poorly written outliers. This linear separability has a number of useful consequences: very few training samples are needed; all points inside the convex hull of the samples correspond to recognizable characters of the same class; thus linear homotopies between points of a class remain in the class. We have used these properties to develop confidence measures based on distances to SVM separating planes and to the convex hull of k nearest neighbours [3]. We have also used homotopies to track special points on characters, allowing baseline determination form characters (as is useful in mathematical handwriting recognition), as opposed to *vice versa* [5].

Additional Directions For recognition of rotated characters or characters subjected to shear, we may represent integral invariants, rather than the (x, y) coordinates, in an orthogonal polynomial basis [2]. Because samples can be represented compactly, and only a few are needed for each character class, it is possible to rapidly adapt to a user's handwriting style as feedback is obtained from correct and incorrect recognitions. Moreover, it is straightforward to find an average example for each character in a user's handwriting style and perform automatic handwriting neatening.

Conclusion We have found that it is quite practical to recognize similar plane curves represented using orthogonal polynomial bases, and that the necessary operations can be performed without leaving that representation.

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