Coincidence of extendible ideals with their minimal kernel.

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Multilinear Operators

Let $E_1, \ldots, E_n, F$ be Banach spaces over $\mathbb{C}$.

- We denote by $\mathcal{L}(E_1, \ldots, E_n; F)$ to the **space of continuous n-linear operators** $T : E_1 \times \cdots \times E_n \to F$ provides with the supremum norm $||T|| = \sup_{x_j \in B_{E_j}} ||T(x_1, \ldots, x_n)||$.

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- If $n = 1$, $\mathcal{L}(E; F)$ is the classical **space of continuous linear operators**.
- We write $\mathcal{L}(E_1, \ldots, E_n)$ if $F = \mathbb{C}$.
- We write $\mathcal{L}(^nE; F)$, when $E_1 = \cdots = E_n = E$.

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$$\|R \circ T \circ (S_1, \dots, S_n)\|_{\mathfrak{A}} \leq \|R\| \cdot \|T\|_{\mathfrak{A}} \cdot \|S_1\| \cdots \|S_n\|.$$
3. The n-linear mapping given by $(\lambda_1, \dots, \lambda_n) \mapsto \lambda_1 \cdots \lambda_n$ belongs to $\mathfrak{A}(^n\mathbb{C})$ and has norm 1.

- If $n = 1$, $(\mathfrak{A}, \|\cdot\|)$ is a **Banach operator ideal**.
- If $F = \mathbb{C}$, $(\mathfrak{A}, \|\cdot\|)$ is a **Banach ideal of n-linear forms**.

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- $\mathcal{L}(E_1, \ldots, E_n; F) :=$ **continuous n-linear operators**.

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- $\mathcal{L}(E_1, \ldots, E_n; F) :=$ **continuous n-linear operators**.
- $\mathcal{L}_f(E_1, \ldots, E_n; F) :=$ **finite type n-linear operators**, those of the form

$$T(x_1, \ldots, x_n) = \sum_{j=1}^{N} x'_{1,j}(x_1) \cdots x'_{n,j}(x_n) \cdot f_j,$$

where $x'_{k,j} \in E'_k$ and $f_j \in F$.

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- $\mathcal{L}_{app}(E_1, \ldots, E_n; F) :=$ **approximable n-linear operators**, the ones that can be approximated by finite type n-linear operators.
  More precisely, $\mathcal{L}_{app} = \overline{\mathcal{L}_f}^{\|\cdot\|}$.

Tensor products and tensor norms

We denote by $\otimes_{j=1}^{n} E_j$ the n-**fold tensor product** and by

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We say that α is a **tensor norm** of order n if α assigns to the normed spaces $E_1, \ldots, E_n$ a norm $\alpha(\cdot \, , \otimes_{j=1}^{n} E_j)$ on the n-fold tensor product $\otimes_{j=1}^{n} E_j$ such that

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We denote by $(\otimes_{j=1}^{n} E_j; \alpha)$ the tensor product $\otimes_{j=1}^{n} E_j$ endowed with the norm $\alpha(\cdot , \otimes_{j=1}^{n} E_j)$, and we write $(\widetilde{\otimes}_{j=1}^{n} E_j; \alpha)$ for its completion.

Tensor products and tensor norms

A tensor norm α is **finitely generated** if for every normed spaces $E_1, \ldots, E_n$ and $z \in \otimes_{j=1}^n E_j$ we have

$$\alpha(z, \otimes_{j=1}^n E_j) := \inf \left\{ \alpha(z, \otimes_{j=1}^n M_j) : \ M_j \in FIN(E_j), \ z \in M_1 \otimes \cdots \otimes M_n \right\}.$$

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If $\mathfrak{A}$ is a vector-valued ideal of multilinear operators, its **associated tensor norm** is the unique finitely generated tensor norm α, of order $n+1$, satisfying

$$\mathfrak{A}(M_1, \ldots, M_n; N) \overset{1}{=} (M_1' \otimes \cdots \otimes M_n' \otimes N; \alpha)$$

for every finite dimensional spaces $M_1, \ldots, M_n, N$.

In that case we write that $\mathfrak{A} \sim \alpha$.

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For example, $\mathcal{L} \sim \varepsilon$, $\mathcal{L}_{app} \sim \varepsilon$, $\mathcal{N} \sim \pi$, $P\mathcal{I} \sim \pi$ and $G\mathcal{I} \sim \pi$.

Minimal ideals

The minimal kernel of $\mathfrak{A}$ is defined as the composition ideal

$$\mathfrak{A}^{min} := \overline{\mathfrak{F}} \circ \mathfrak{A} \circ (\overline{\mathfrak{F}}, \ldots, \overline{\mathfrak{F}}),$$

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The $\mathfrak{A}$-minimal norm of T_1 is given by

$$\|T_1\|_{\mathfrak{A}^{min}} := \inf \left\{ \|S\| \cdot \|T_2\|_{\mathfrak{A}} \cdot \|R_1\| \cdots \|R_n\| \right\},$$

where the infimum runs over all possible factorizations $T_1 = S \circ T_2 \circ (R_1, \dots, R_n)$ as above.

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The ideal $\mathfrak{A}$ is said to be minimal if $\mathfrak{A}^{min} = \mathfrak{A}$.

Minimal ideals - Representation theorem

The following theorem due to Defant and Floret shows a close relation between the tensor product and the minimal kernel of an ideal.

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Representation theorem for minimal ideals

Let $E_1, \dots, E_n, F$ be Banach spaces and let $\mathfrak{A} \sim \alpha$ be a minimal ideal. Then there is a natural quotient mapping

$$(E_1' \tilde{\otimes} \dots \tilde{\otimes} E_n' \tilde{\otimes} F; \alpha) \overset{1}{\twoheadrightarrow} \mathfrak{A}(E_1, \dots, E_n; F)$$

defined on $E_1' \otimes \cdots \otimes E_n' \otimes F$ by the obvious rule

$$\sum_{j=1}^r (x_1^j)' \otimes \cdots \otimes (x_n^j)' \otimes f_j \mapsto \sum_{j=1}^r (x_1^j)'(\cdot) \dots (x_n^j)'(\cdot) f_j.$$

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Therefore, we have $(E_1' \tilde{\otimes} \ldots \tilde{\otimes} E_n' \tilde{\otimes} F; \alpha) \overset{1}{\twoheadrightarrow} \mathfrak{A}^{min}(E_1, \ldots, E_n; F)$ for any ideal $\mathfrak{A}$ associated to α.

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Moreover, if $E_1', \dots, E_n', F$ have the bounded approximation property, then $(E_1' \tilde{\otimes} \dots \tilde{\otimes} E_n' \tilde{\otimes} F; \alpha) \overset{1}{=} \mathfrak{A}^{min}(E_1, \dots, E_n; F)$.

Motivation of the problem

Recall that a Banach space E has a **Schauder basis** $(e_k)_{k\in\mathbb{N}}$ if there are coordinate functionals $(e'_k)_{k\in\mathbb{N}}$ such that every vector x is written as $x = \sum_{k=1}^{\infty} e'_k(x)e_k$.

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Let $T \in \mathcal{L}(E_1, \ldots, E_n; F)$, then

$$T(x_1, \ldots, x_n) = \sum_{j_1,\ldots,j_n,l} e'_{j_1}(x_1)\cdots e'_{j_n}(x_n)\cdot f_l.$$

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Remark

Let $E_1, \ldots, E_n$ be Banach spaces with Schauder bases $(e_{j_1})_{j_1}, \ldots, (e_{j_n})_{j_n}$ respectively and α be a tensor norm of order n. There is a natural ordering (called the generalized square ordering of Gelbaum-Gil de Lamadrid) in $\mathbb{N}^n$ such that the monomials

$$(e_{j_1}\otimes\cdots\otimes e_{j_n})_{j_1,\ldots,j_n}$$

form a Schauder basis of $(E_1\tilde{\otimes}\ldots\tilde{\otimes}E_n, \alpha)$.

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If $E'_1, \ldots, E'_n$ and F have bases $(e'_{j_1})_{j_1}, \ldots, (e'_{j_n})_{j_n}, (f_l)_l$ respectively, when the monomials $(e'_{j_1}(\cdot) \cdots e'_{j_n}(\cdot) \cdot f_l)_{j_1,\ldots,j_n,l}$ (ordered in some way) **form a basis** of $\mathfrak{A}(E_1, \ldots, E_n; F)$?

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Let $\mathfrak{A} \sim \alpha$ be an ideal of n-linear operators.
The representation theorem for minimal ideals gives a natural norm one inclusion from $(E_1' \tilde{\otimes} \ldots \widetilde{\otimes} E_n' \tilde{\otimes} F; \alpha)$ to $\mathfrak{A}(E_1, \ldots, E_n; F)$

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Representation theorem Natural inclusion

Our goal is to find conditions under which the mapping ϱ results a metric surjection.

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Representation theorem — Natural inclusion

Our goal is to find conditions under which the mapping ϱ results a metric surjection. In that case we get that there is an isometric isomorphism between $\mathfrak{A}^{min}$ and $\mathfrak{A}$ (coincidence result).

Motivation of the problem

If $E_1', \ldots, E_n'$ and F have bases $(e'_{j_1})_{j_1}, \ldots, (e'_{j_n})_{j_n}, (f_l)_l$ respectively, when the monomials $(e'_{j_1}(\cdot) \cdots e'_{j_n}(\cdot) \cdot f_l)_{j_1,\ldots,j_n,l}$ (ordered in some way) **form a basis** of $\mathfrak{A}(E_1, \ldots, E_n; F)$?

Let $\mathfrak{A} \sim \alpha$ be an ideal of n-linear operators.
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Then $\mathfrak{A}$ inherits properties of the tensor product (such as having basis).

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- α has the Radon-Nikodým property if and only if $\alpha/$ has it.
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- In the proof of Lewis theorem it is used that if $\mathcal{A}^{min}(E; \ell_1) \overset{1}{=} \mathcal{A}(E; \ell_1)$, then $\mathcal{A}^{min}(E; \ell_1(J)) \overset{1}{=} \mathcal{A}(E; \ell_1(J))$ for all index set J.

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Again, using the representation theorems, we have that if $\mathcal{U}$ is a maximal ideal of n-linear forms associated to α, then

$$\mathcal{U}^{min}({}^n c_0) \stackrel{1}{=} \mathcal{U}({}^n c_0).$$

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Carando-Galicer theorem

Let α be a tensor norm with the symmetric Radon-Nikodým property and $E_1, \dots, E_n$ be Asplund spaces. Then

$$(\widetilde{\otimes}_{i=1}^n E_i', \backslash\alpha/) \twoheadrightarrow (\widetilde{\otimes}_{i=1}^n E_i, /\alpha'\backslash)'$$

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- If $\mathcal{U} \sim \alpha$, then $\mathcal{U}_{(/\alpha'\backslash)} \sim /\alpha'\backslash$ and every n-linear form $T \in \mathcal{U}_{(/\alpha'\backslash)}(E_1, \ldots, E_n)$ can be extended in each variable with the same ideal norm.

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- In the proof of the C-G theorem it is used that if $\mathcal{U}^{min}({}^n c_0) \stackrel{1}{=} \mathcal{U}({}^n c_0)$, then $\mathcal{U}^{min}(c_0(J_1), \ldots c_0(J_n)) \stackrel{1}{=} \mathcal{U}(c_0(J_1), \ldots, c_0(J_n))$ for all index sets $J_1, \ldots, J_n$.

New Radon-Nikodým property

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Radon-Nikodým property for ideals of multilinear operators

Let $\mathfrak{A} \sim \alpha$ be an ideal of n-linear operators and F be a Banach space. We say that $\mathfrak{A}$ has the F-Radon-Nikodým property (F-RNp) if

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This definition says that if $\mathfrak{A}$ has the F-RNp then

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If F contains no copy of c_0 or $\mathfrak{A} \subseteq \mathcal{L}_{wsc}$ (weakly sequentially continuous n-linear operators), then $\mathfrak{A}$ has the F-RNp.

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- An ideal of multilinear operators $\mathfrak{A}$ is **extendible** if for every $G_1 \supseteq E_1, \ldots, G_n \supseteq E_n, F$ and every $T \in \mathfrak{A}(E_1, \ldots, E_n; F)$, there exists an extension $\widetilde{T} \in \mathfrak{A}(G_1, \ldots, G_n; F)$ with $\|T\|_{\mathfrak{A}} = \|\widetilde{T}\|_{\mathfrak{A}}$.

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- For $1 \leq k \leq n$, the k**-Arens extension** of T, Ext_k, is the canonical extension to the bidual in the k-coordinate:

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- Let $\mathfrak{A}$ be an ideal of multilinear operators, we say that $\mathfrak{A}$ is an ***F*-Arens stable ideal** if the mapping

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  Note that the condition above says that the range of every Arens extension remains on F.

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Note that the conditions of the Main theorem I are a bit more general than those of Lewis theorem (for linear operators) and C-G theorem (for multilinear forms).

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Main theorem II

Let $\mathfrak{A}$ be an F''-Arens stable extendible ideal with the F''-RNp. If $E_1, \ldots, E_n$ have shrinking bases and F'' has the bounded approximation property,

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- In other hand, as we are interested in searching for monomial bases on spaces of multilinear operators, it is natural to deal with spaces which have shrinking Schauder bases.

Main theorem II

Let $\mathfrak{A}$ be an F''-Arens stable extendible ideal with the F''-RNp. If $E_1, \ldots, E_n$ have shrinking bases and F'' has the bounded approximation property, then

$$(E_1' \tilde{\otimes} \ldots \tilde{\otimes} E_n' \tilde{\otimes} F, \alpha) \overset{1}{=} \mathfrak{A}^{min}(E_1, \ldots, E_n; F) \overset{1}{=} \mathfrak{A}(E_1, \ldots, E_n; F).$$

Consequences

As a consequence of the main theorems, we have

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Let $\mathfrak{A} \sim \alpha$ be an extendible ideal of n-linear operators.

1. If $\mathfrak{A}$ is F-Arens stable, has the F-RNp and $E_1', \dots, E_n', F$ have Schauder bases $(e_{j_1}')_{j_1}, \dots, (e_{j_n}')_{j_n}, (f_l)_l$ respectively,

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2. If $\mathfrak{A}$ is F''-Arens stable and has the F''-RNp, F'' has the bounded approximation property, $E_1, \ldots, E_n$ have shrinking Schauder bases $(e_{j_1}')_{j_1}, \ldots, (e_{j_n}')_{j_n}$ respectively and F has basis $(f_l)_l$,

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Applications for the ideal of extendible n-linear operators

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A multilinear operator $T : E_1 \times \cdots \times E_n \to F$ is **extendible** if for every Banach spaces $G_1, \ldots, G_n$ containing $E_1, \ldots, E_n$ respectively, there exists $\widetilde{T} \in \mathcal{L}(G_1, \ldots, G_n; F)$ extending T. We denote by $\mathcal{E}(E_1, \ldots, E_n; F)$ the space of all extendible multilinear operators and results an ideal of multilinear operators with the norm

$$||T||_{\mathcal{E}} := \inf\{c > 0 : \quad \text{for every } \; G_i \supset E_i \; \text{ there exists an extension } \; \widetilde{T} \; \text{ of } \; T \; \text{ with norm } \; \leq c\}.$$

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The ideal $\mathcal{E}$ is extendible and F'-Arens stable for every dual space F'.

In addition, if G is a Banach space which contains no copy of c_0, then $\mathcal{E}$ has the G-RNp.

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Applying the Main theorem I we obtain the next corollary.

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Applications for the ideal of extendible n-linear operators

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3. If F' is a dual space which contains no copy of c_0, and $E_1, \ldots, E_n; F$ have shrinking Schauder bases $(e_{j_1})_{j_1}, \ldots, (e_{j_n})_{j_n}, (f_l)_l$ respectively,

Applications for the ideal of extendible n-linear operators

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3. If F' is a dual space which contains no copy of c_0, and $E_1, \ldots, E_n; F$ have shrinking Schauder bases $(e_{j_1})_{j_1}, \ldots, (e_{j_n})_{j_n}, (f_l)_l$ respectively, then the monomials (associated to the coordinate functionals)

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Note that Corollary I shows results of coincidence and existence of Schauder basis for the ideal of extendible multilinear operators where the range space is a dual space.

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Applying the Main theorem II we obtain a corollary for the ideal of extendible multilinear operators for any range space.

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In particular,
if F has also a basis then the monomials with the generalized square ordering form a basis of $\mathcal{E}(E_1, \dots, E_n; F)$.

Other applications

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- The main theorems permit to relate structural properties of the ideal $\mathfrak{A}$ with properties of the spaces involved and their tensor product such as separability, the Radon-Nikodým and Asplund properties.

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  If $E_1, \ldots, E_n$ are Asplund spaces, then
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- These results hold for ideals of homogeneous polynomials too.

THANKS!!!!!