Banach-Stone theorems for algebras of germs of holomorphic functions.

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Banach: metric spaces. Stone: topological spaces. Arens and Kelly, 1947: complex case

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(Bachir 2001) Let X and Y be complete metric spaces, let E e and F be Banach spaces with property (*smooth*). Then $\mathcal{C}^*(X, E)$ and $\mathcal{C}^*(Y, F)$ are isometric if, and only if, X and Y are homeomorphic.

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Proof is based on the spectra of the algebras $\mathcal{C}(K)$ and $\mathcal{C}(L)$.

Algebras of differentiable functions

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The results listed here are presented in the article *Variations on the Banach-Stone theorem* [3].

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which is complete but not metrizable when $\dim E = \infty$.

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Which means that there exists an holomorphic bijective mapping $\varphi : U \longrightarrow V$ such that φ^{-1} is holomorphic.

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We denote by $\mathcal{H}_b(U)$ the algebra of all $f \in \mathcal{H}(U)$ that are bounded on each U-bounded subset. Endowed with the topology of the uniform convergence on the sets $(U_n)_{n\in\mathbb{N}}$, we have that $\mathcal{H}_b(U)$ is a Fréchet algebra.

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1. There exists an algebra topological isomorphism $T : \mathcal{H}_b(U) \longrightarrow \mathcal{H}_b(V)$.
2. There exists a bijective mapping $\varphi \in \mathcal{H}_b(\widehat{U}_{\mathcal{P}(E)}; \widehat{V}_{\mathcal{P}(F)})$ such that $\varphi^{-1} \in \mathcal{H}_b(\widehat{V}_{\mathcal{P}(F)}; \widehat{U}_{\mathcal{P}(E)})$, with

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that each $f \in \mathcal{H}_b(U)$ admits an extension $\tilde{f} \in \mathcal{H}_b(\widehat{U}_{\mathcal{P}(E)})$.

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Let $f_1,\ f_2 \in h(K)$ and let $U_1,\ U_2$ be open subsets of E, with $K \subset U_1$ and $K \subset U_2$, such that $f_1 \in \mathcal{H}(U_1)$ e $f_2 \in \mathcal{H}(U_2)$.

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We say that f_1 and f_2 are *equivalent* (and we denote $f_1 \sim f_2$) if there exists an open subset $W \subseteq E$ with $K \subset W \subseteq U_1 \cap U_2$ and such that $f_1 = f_2$ em W.

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Then $\sim$ is an equivalence relation in $h(K)$ and we denote $\mathcal{H}(K) = h(K)/\sim$.

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We observe that two compact sets are *biholomorfically equivalent* if there exists a biholomorphic mapping between open subsets containing each of the compact sets.

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$$\begin{array}{ccc} \mathcal{H}(K) & \xrightarrow{T} & \mathcal{H}(L) \\ {\scriptstyle i_k}\Big\uparrow & & \Big\uparrow{\scriptstyle i_{m_k}} \\ \mathcal{H}_b(U_k) & \xrightarrow{T_k} & \mathcal{H}_b(V_{m_k}) \end{array}$$

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Applying the same argument for $S = T^{-1}$, for m_k we find $n_k > k$, a continuous homomorphism

$S_k : \mathcal{H}_b(V_{m_k}) \longrightarrow \mathcal{H}_b(U_{n_k})$ and a holomorphic mapping
$\psi_k : (\widehat{U_{n_k}})_{\mathcal{P}(E)} \longrightarrow (\widehat{V_{m_k}})_{\mathcal{P}(F)}$,

such that $\widetilde{S_k(g)} = \tilde{g} \circ \psi$, for all $g \in \mathcal{H}_b(V_{m_k})$.

Using Theorem 7, there exists a holomorphic mapping

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Then we prove that $\varphi_k \circ \psi_k : (\widehat{U_{n_k}})_{\mathcal{P}(E)} \longrightarrow (\widehat{U_k})_{\mathcal{P}(F)}$ is the inclusion mapping, for all $k \in \mathbb{N}$.

Next it follows that $\varphi_1 = \varphi_k$ on each $(\widehat{V_{m_k}})_{\mathcal{P}(F)}$,

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The implication (2) $\Rightarrow$ (1) is not difficult to prove, and is true with no assumptions on E and F, and for K and L balanced compact subsets.

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Corollary

Let E and F Tsirelson-like Banach spaces, let $K \subset E$ and $L \subset F$ be balanced and polynomially convex compact subsets. Then the following conditions are equivalent:

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Corollary 9 improves a result of [6]. Where the results are obtained when $K = \widehat{K}_{\mathcal{P}(^mE)}$ and $L = \widehat{L}_{\mathcal{P}(^nF)}$.

THANK YOU!

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