Bounded holomorphic functions attaining their norms in the bidual

WidaBA14

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(joint work with Dani and Silvia)

UBA

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A little of history...

X, Y Banach spaces.
B_X = "closed unit ball".
$X' = \mathcal{L}(X; \mathbb{K})$ the dual space of X.
$NA\mathcal{L}(X; \mathbb{K})$ = "norm attaining functionals".
$NA \cdots$ = "norm attaining functions in $\cdots$".

James ($\sim$'50)

$NA\mathcal{L}(X; \mathbb{K}) = X'$ if and only if X is reflexive.

Bishop-Phelps ('61)

$NA\mathcal{L}(X; \mathbb{K})$ is dense in $\mathcal{L}(X; \mathbb{K}) = X'$.

Natural question: what can we say for the space $\mathcal{L}(X; Y)$ of linear operators?

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There is no Bishop-Phelps theorem in $\mathcal{L}(X;Y)$ (Lindenstrauss ('63)).

$$NA\mathcal{L}_0(X;Y) = \{T \in \mathcal{L}(X;Y) : \quad T'' \in NA\mathcal{L}(X'';Y'')\}$$

$$NA\mathcal{L}(X;Y) \subseteq NA\mathcal{L}_0(X;Y)$$

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Problems of interest

- Find spaces for which the Bishop-Phelps holds (linear, multilinear, polynomial, holomorphic cases).
- Cuantitative versions (*i.e.* Bishop- Phelps-Bollobás results).
- Possible extensions of Lindenstrauss theorem to multilinear, polynomial and holomorphic cases.

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Plan of the talk

1. **Lindenstrauss-type theorems.**
   - Some known results.
   - Partial results for polynomials and holomorphic functions.
2. **Counterexamples to Bishop-Phelps.**
   Spaces for which the Bishop-Phelps theorem fails, but the Lindenstrauss theorem holds.
3. **Lindenstrauss for some classes of multilinear mappings.**

Lindenstrauss-type theorems

Multilinear case

$\mathcal{L}(^N X_1 \times \cdots \times X_N; Y) =$ "$\Phi : X_1 \times \cdots \times X_N \to Y$, N-linear mappings".

Arens extension ("bitranspose")

Given $\Phi \in \mathcal{L}(^N X_1 \times \times X_N; Y)$ define $\overline{\Phi} \in \mathcal{L}(^N X_1'' \times \times X_N''; Y'')$ by

$$\overline{\Phi}(z_1, \ldots, z_N) = w^* - \lim_{\alpha_1} \cdots \lim_{\alpha_N} \Phi(x^1_{\alpha_1}, \ldots, x^N_{\alpha_N})$$

where $x^i_{\alpha_i} \xrightarrow{w^*} z_i$ if $\alpha_i \in \Lambda_i$.

$$NA\mathcal{L}_0(^N X_1 \times \cdots \times X_N; Y) = \{\Phi : \quad \overline{\Phi} \quad \text{is norm attaining}\}.$$

Multilinear Lindenstrauss (Acosta-García-Maestre ('06))

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Polynomial case

$\mathcal{P}(^N X;Y) =$ "$P : X \to Y$, N-homogeneous polynomial".

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Given $P \in \mathcal{P}(^N X;Y)$ define $\overline{P} \in \mathcal{P}(^N X'';Y'')$,

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where Φ is the unique symmetric N-linear mapping associated.

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2-homogeneous Lindenstrauss (Aron-García-Maestre ('02))

$$NA\mathcal{P}_0(^2 X;Y) \text{ is dense in } \mathcal{P}(^2 X;Y).$$

(vector-valued case: Choi-Lee-Song ('10))

Extensions to the N-homogeneous case?

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Theorem (Carando-Lassalle-M. ('12))

Suppose X' is separable and has the approximation property. Then $NA\mathcal{P}_0(^N X; Y')$ is dense in $\mathcal{P}(^N X; Y')$.

Idea of the proof (scalar case)

$$\mathcal{P}(^N X) \overset{1}{=} \left(\tilde{\otimes}^{N,s}_{\pi_s} X\right)'$$

$$\begin{tikzcd} x \arrow[d] \\ x \otimes \cdots \otimes x \end{tikzcd} \qquad \begin{tikzcd} X \arrow[r, "P"] \arrow[d] & \mathbb{K} \\ \otimes^{N,s}_{\pi_s} X \arrow[ur, "L_P"'] & \end{tikzcd}$$

- $u \in \left(\otimes^{N,s}_{\pi_s} X\right) \Rightarrow u = \sum_{i=1}^{m} \lambda_i (x_i \otimes \cdots \otimes x_i)$ with $x_i \in B_X$.
- $\pi_s(u) = \text{ínf}\left\{\sum_{i=1}^{m} |\lambda_i|\right\}$ (infimum over all representations).
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Take $P \in \mathcal{P}(^N X)$ and $\varepsilon > 0$.

Linearize and use Bishop-Phelps:

- $\|L_Q - L_P\| = \|Q - P\| < \varepsilon$.
- $|L_Q(u_0)| = \|L_Q\|$ for $u_0 \in B_{\tilde{\otimes}^{N,s}_{\pi_s} X}$.

$$\|\overline{Q}\| = |L_Q(u_0)| = \left| \int_{B_{X''}} \overline{Q}(z) d\mu_{u_0}(z) \right| \leq \|\overline{Q}\| \|\mu_{u_0}\| \leq \|\overline{Q}\|$$

(hypothesis on X!!)

Then, there exists $Q \in \mathcal{P}(^N X)$ such that:

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$\mathcal{P}_k(X;Y) =$ "polynomials $P : X \to Y$ of degree less than or equal to k".

$P = \sum_{j=0}^k P_j$ with $P_j \in \mathcal{P}({}^jX;Y)$, $\overline{P} = \sum_{j=0}^k \overline{P_j}$.

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The Lindenstrauss theorem holds in $\mathcal{P}_k(X;Y')$.

$$\mathcal{A}_u(B_X;Y) = \text{“}f : B_X \to Y \text{ unif. cont. } / \ f|_{B_X^\circ} \text{ holomorphic”}$$

Corollary

The Lindenstrauss theorem holds in $\mathcal{A}_u(B_X;Y')$.

Following ideas of Choi and Kim, if Y has property (β) of Lindenstrauss (*e.g.* c_0, ℓ_∞) then the Lindenstrauss theorem holds in $\mathcal{P}(^N X;Y)$, $\mathcal{P}_k(X;Y)$ and $\mathcal{A}_u(B_X;Y)$.

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Counterexamples to Bishop-Phelps

Preduals of Lorentz

Let $w = (w_i)_i$ admissible sequence ($w_1 = 1$, $w_i \searrow 0$, $w \in c_0 \backslash \ell_1$).

$$d_*(w,1) = \left\{ (x(i))_i : \quad \lim_n \frac{\sum_{i=1}^n x(i)^*}{W(n)} = 0 \right\}$$

$$\|(x(i))_i\|_W = \sup_n \frac{\sum_{i=1}^n x(i)^*}{W(n)}$$

where $W(n) = \sum_{i=1}^n w_i$ y $(x(i)^*)_i$ is the decreasing rearrangement of $(|x(i)|)_{i \in \mathbb{N}}$.

Important properties

- The lack of extreme points of $B_{d_*(w,1)}$.
- If $w \in \ell_p$ $(1 < p < \infty)$ then $d_*(w,1) \hookrightarrow \ell_p$.

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Fix $w \in \ell_p$ $(1 < p < \infty)$,

- **There is no** Bishop-Phelps in $\mathcal{P}(^N d_*(w,1))$ if $N \geq p$.
- **There is no** Bishop-Phelps in $\mathcal{P}(^N d_*(w,1); \ell_p)$ $\forall N \in \mathbb{N}$.

(Jiménez S.-Payá ('96)/Carando-Lassalle-M. ('12))

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- There is a Banach space Z such that **there is no** Bishop-Phelps in $\mathcal{A}_u(B_{c_0}; Z'')$ (no counterexample in the scalar case!!).

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We also obtain counterexamples for polynomials with values in c_0 (which has property (β)).

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Symmetric multilinear mappings.
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Theorem (Carando-Lassalle-M. ('14))

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Ideals of multilinear mappings.

Theorem (Acosta-García-Maestre ('06))

Let $\mathcal{U} = \mathcal{I}$ or $\mathcal{N}$. Then the set of N-linear operators in $\mathcal{U}(X_1 \times \cdots \times X_N)$ such that their Arens extensions attain the supremum-norm is $\|\cdot\|_{\mathcal{U}}$-dense in $\mathcal{U}(X_1 \times \cdots \times X_N)$.

Following carefully their proof, we demonstrate the same for *stable* ideals $\mathcal{U}(X_1 \times \cdots \times X_N)$.

What is stable?

In the symmetric case, for all $\mathbf{a} = (a_1, \ldots, a_N) \in X_1 \times \cdots \times X_N$ and all $1 \le j \le N$,

$$\Phi_{j,\mathbf{a}}(\mathbf{x}) = \Phi(x_1, \ldots, x_j, a_{j+1}, \ldots, a_N)\Phi(a_1, \ldots, a_j, x_{j+1}, \ldots, x_N)$$

belongs to $\mathcal{U}(X_1 \times \cdots \times X_N)$ with some control on the norm.

Ideals of multilinear mappings.

Theorem (Acosta-García-Maestre ('06))

Let $\mathcal{U} = \mathcal{I}$ or $\mathcal{N}$. Then the set of N-linear operators in $\mathcal{U}(X_1 \times \cdots \times X_N)$ such that their Arens extensions attain the supremum-norm is $\| \cdot \|_{\mathcal{U}}$-dense in $\mathcal{U}(X_1 \times \cdots \times X_N)$.

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For instance, take $N = 4$ and $j = 2$. In this case,

$$(x_1, x_2, x_3, x_4) \mapsto \Phi(x_1, x_2, a_3, a_4)\, \Phi(a_1, a_2, x_3, x_4)$$

belongs to $\mathcal{U}(X_1 \times \cdots \times X_4)$, for any $(a_1, \cdots, a_4)$ with some control on the norm.

Examples: nuclear, integral, extendible, multiple p-summing, ...

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