Advanced Workshop ELGA 2011

CIMPA-ICTP-UNESCO-MICINN School on Algebraic Geometry and Applications

ABSTRACTS - August 8-12, 2011

On Fano foliations

Carolina Araujo

In this talk I shall discuss *Fano foliations* on complex projective manifolds. I will concentrate on the special class of *Del Pezzo* foliations. As I shall explain, such foliations have algebraic and rationally connected leaves, except for some well understood degree 1 foliations on $\mathbb{P}^n$. I will also discuss the classification of Del Pezzo foliations having mild singularities. This is a joint work with Stéphane Druel.

Implicit equations of rational hypersurfaces by means of free resolutions

Nicolás Botbol

I will overview a method for computing the implicit equation of a hypersurface given as the image of a rational map $f : X \dashrightarrow \mathbb{P}^n$; first, when X is $\mathbb{P}^{n-1}$, or an aCM projective toric variety, then, X is a product of projective spaces. This technique is based on elimination theory by means of homological algebra. As noticed by Jouanolou, Hurwitz proved in 1913 that, for n generic homogeneous polynomials in $r \geq n$ variables, the Koszul complex is acyclic in positive degrees. It was known around 1930 that resultants may be calculated as a Mc Rae invariant of this complex. This expresses the resultant as a determinant of this complex. In 2002, Jouanolou and Busé also noticed that this machinery could be used for computing implicit equations of hypersurfaces defined as the image of a rational map $f : \mathbb{P}^{n-1} \dashrightarrow \mathbb{P}^n$. The main difficulty comes from the nature of the base points, that in many cases make the theory no applicable.

I will sketch some alternatives to avoid base points, based on choosing a projective toric compactification of the domain, that is better suited to the nature of the map f, as well as studying the implicitization problem directly, without an embedding in a projective space. We use the multihomogeneous structure of the coordinate ring of X, and we adapt the method of loc. cit. to this setting.

On the geometry of the moduli space of vector bundles on curves

Leticia Brambila Paz

Moduli spaces of vector bundles have been studied from different points of view. In this talk I will recall some of the basic properties of moduli spaces. In particular I will recall the construction of the Hecke curves and see how it can be generalized to subvarieties of higher dimension. This is joint work with Osbaldo Mata.

On Lê cycles and Milnor classes

Roberto Callejas-Bedregal

The purpose of this talk is to establish a link between the theory of Chern classes for singular varieties and the geometry of the varieties in question. Namely, we show that if Z is a compact hypersurface in some complex manifold, defined by a holomorphic section of a very ample line bundle, then its Milnor classes, regarded as elements in the Chow group of Z, determine the global Lê cycles of Z, and viceversa: the Lê cycles determine the Milnor classes. Morally this implies, among other things, that the Milnor classes determine the topology of the local Milnor fibres at each point of Z, and the geometry of the local Milnor fibres determines the corresponding Milnor classes.

Near weight functions and Groebner basis for the defining ideal of affine curves

Cícero Carvalho

In 1998 Høholdt, van Lint and Pellikaan (see [2]) introduced the concept of a "weight function" defined on a $\mathbb{F}$-algebra ($\mathbb{F}$ a field) and used it to construct linear codes and find bounds for their minimum distances, studying the case where $\mathbb{F}$ is a finite field. Later, this concept was generalized to that of "near weight functions" and Carvalho and Silva ([1]) characterized the $\mathbb{F}$-algebras which admit a so-called complete set of m near weight functions as being the rings of regular functions of irreducible affine curves defined over $\mathbb{F}$ and having a total of exactly m branches at the points at infinity. In this talk we show that starting from such a ring of regular functions and using a subsemigroup of $\mathbb{N}_0^m$ obtained from the set of m near weight functions we may determine a reduced Groebner basis for the defining ideal of the ring. Conversely, given a suitable subsemigroup of $\mathbb{N}_0^m$ and an associated basis B of an ideal I in a polynomial ring A, we prove that if B is a Groebner basis for I then A/I is the ring of regular functions of an affine curve having m branches at infinity. These results generalize results from Matsumoto and Miura ([3]) which correspond to the case $m = 1$.

References

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The standard conjectures for some holomorphic symplectic varieties

François Charles

Holomorphic symplectic varieties are smooth simply connected projective varieties which carry a unique everywhere nondegenerate two-form. Using techniques from hyperkähler geometry, we prove Grothendieck's standard conjectures in the theory of algebraic cycles for those varieties which are deformations of Hilbert schemes of points on $K3$ surfaces. This is joint work with Eyal Markman.

Abel maps and Brill-Noether varieties

Juliana Coelho

Let C be an algebraic curve. For given integers r and d, the variety parameterizing line bundles of degree d on C whose space of global sections has dimension at least $r + 1$ is denoted by $W_d^r(C)$ and is called a Brill-Noether variety. These varieties are related to the degree-d Abel maps taking d points of C to the line bundle associated to the divisor given by them. In this talk we present some results concerning Brill-Noether varieties for singular curves.

Implicitization of surfaces via geometric tropicalization

Maria Angelica Cueto

In this talk we discuss recent developments in tropical methods for implicitization of surfaces. This study was pioneered in the generic case by work of Sturmfels, Tevelev and Yu, and is based on the theory of geometric tropicalization, developed by Hacking, Keel and Tevelev, The latter hinges on computing the tropicalization of subvarieties of tori by analyzing the combinatorics of their boundary in a suitable (tropical) compactification. We enhance this theory by providing a formula for computing weights on tropical varieties, a key tool for tropical implicitization. Finally, we address the question of tropical implicitization for non-generic surfaces and illustrate our techniques with several numerical examples in 3-space.

Compatible systems of Galois representations with generically large images and applications

Luis Victor Dieulefait

We will discuss in this talk recent joint work with Gabor Wiese on the construction of compatible systems of Galois representations attached to modular forms with certain local types such that the images are "as large as possible" for every prime (note that by a result of Ribet, for any non-CM cuspidal modular form it is known that the exceptional set of primes where the image fails to be maximal is finite: we are proving that in certain cases this set is empty). We will explain how to prove some new results on the Inverse Galois Problem for linear groups over finite fields of arbitrary exponent using similar techniques. Finally, we will explain a recent joint work with Gabor Wiese and Sara Arias de Reyna where we construct symplectic compatible systems of Galois representations (of any even dimension) attached to automorphic forms with certain local types such that we can prove that the residual images are as large as possible for almost every prime (i.e., for all but finitely many primes).

Polyhedral Adjunction Theory

Sandra Di Rocco

Adjunction linear systems have classically played an important role in the classification of projective, normal and Q-Gorenstein algebraic varieties. As projective toric varieties are associated to convex integral polytopes there is an induced adjunction for polytopes, which we call "Polyhedral adjunction theory".
Classical geometrical invariants as the nef-value and the canonical threshold of a line bundle have natural correspondent convex invariants that play an important role in the classification of convex polytopes and in the characterization of special classes of polytopes like Cayley sums. I will report on joint work with C. Haase, B. Nill and A. Paffenholz.

On the last Hilbert-Samuel coefficient

Juan Elias

In 1978 Lipman presented a proof of the existence of a desingularization for any excellent surface. The strategy of Lipman's proof is based on the finiteness of the number $H(R)$, defined as the supreme of the second Hilbert-Samuel coefficient $e_2(I)$, where I range the set of normal $\mathbf{m}$-primary ideals of a Noetherian complete local ring $(R, \mathbf{m})$. On the other hand Huckaba and Huneke proved that if I is a $\mathbf{m}$-primary ideal of a two dimensional Cohen-Macaulay local ring $(R, \mathbf{m})$ such that I^n is integrally closed for $n \gg 0$, in particular if I is normal, then the associated graded ring of R with respect to I^n is Cohen-Macaulay for $n \gg 0$.
The problem studied in the talk is the extension of the result of Lipman on $H(R)$ to $\mathbf{m}-$primary ideals I of a d-dimensional Cohen-Macaulay ring R such that the associated graded ring of R with respect to I^n is Cohen-Macaulay for $n \gg 0$

Abel maps and limit linear series

Eduardo Esteves

We explore the relationship between limit linear series and fibers of Abel maps in the case of curves with two smooth components glued at a single node. To an r-dimensional limit linear series satisfying a certain exactness property (weaker than the refinedness property of Eisenbud and Harris) we associate a closed subscheme of the appropriate fiber of the Abel map. We then describe this closed subscheme explicitly, computing its Hilbert polynomial and showing that it is Cohen–Macaulay of pure dimension r. We show that this construction is also compatible with one-parameter smoothings. Joint work with Brian Osserman (Davis, USA)

Free structures in division rings.

Renato Fehlberg Júnior

Makar-Limanov's conjecture states that if a skew field D with center k is finitely generated and infinite dimensional over k then D contains a free (noncommutative) k-algebra. We will present techniques by Makar-Limanov and Lorenz to prove this conjecture when D is the skew field of fractions of the skew polinomial ring $L[t; \sigma]$, where L is the function field of a curve over k and σ is a k-automorphism of infinite order of L. Next, we will explain how we intend to extend these techniques in order to prove the conjecture when L is the function field of an abelian variety.

Nash problem for surfaces

Javier Fernández de Bobadilla

In the late 60s Nash proposed to study a natural correspondence between the irreducible components of the arc space centered at the singular set of an algebraic variety and the essential components of a resolution of singularities. In the case of surfaces he predicted that this correspondence would be a bijection. He also suggested to study the question in higher dimension. Since them, appart from Nash original question arc spaces have been intensively studied since they constitute the foundations for motivic integration (work of Kontsevich, Denef, Loeser ...) and have been used as a source of invariants for singularities (Mustata, Ishii, de Fernex, Lazarsfed, Ein ...)
In 2003 a counterexample to Nash question in dimension 4 was found by J. Kollar and S. Ishii. In a recent joint work M. Pe Pereira and the speaker have settled Nash prediction for surfaces in the affirmative. T he proof uses a new technique based on topological arguments. I will explain the main ideas of the proof in the talk.

Finite products of zeta-regularized products

Eduardo Friedman

Regularized products $\widehat{\prod}_m a_m$ are an often useful substitute for divergent products. If the Dirichlet series $f(s) := \sum_m a_m^{-s}$ converges for $\mathrm{Re}(s) \gg 0$ and has an analytic continuation to $s = 0$, one defines $\widehat{\prod}_m a_m := \exp\big(-f'(0)\big)$. It is known, since at least the work of Shintani in the 1970's on special values of abelian L-functions attached to totally real fields, that taking zeta-regularized products does not commute with finite products, *i.e.* in general

$$\widehat{\prod}_m (a_m \cdot b_m) \neq (\widehat{\prod}_m a_m) \cdot (\widehat{\prod}_m b_m).$$

Nonetheless, both sides of this non-equality seem related, since in all known examples their ratio is far simpler than either side. We will discuss the discrepancy F_n given by

$$\exp(F_n) := \frac{\widehat{\prod}_m \big(\prod_{j=1}^n a_{m,j}\big)}{\prod_{j=1}^n \big(\widehat{\prod}_m a_{m,j}\big)}.$$

For a rather general class of products, associated to polynomials P_j in several variables, the discrepancy $F_n(P_1, \ldots, P_n)$ of n products is a sum of pairwise contributions $F_2(P_i, P_j)$. Namely,

$$\Big(\sum_{j=1}^n \deg P_j\Big) F_n(P_1, \ldots, P_n) = \sum_{1 \le i < j \le n} (\deg P_i + \deg P_j) F_2(P_i, P_j).$$

Thus, there are no higher interactions behind the non-commutativity. This provides some insight into Shintani's version of the Kronecker limit formula.
This is joint work with Victor Castillo-Garate.

The Kuznetsov Trace Formula for $GL(n)$

Dorian Goldfeld

In this talk I will present a version of the Kuznetsov trace formula for the group $GL(n)$ with $n > 1$. The new ingredient is a very explicit version of the Lebedev-Whittaker transform which allows one to choose test functions with good properties on both the spectral and geometric sides of the trace formula. Some applications will be given.

Refined curve counting on algebraic surfaces

Lothar Göttsche

Let L be ample line bundle on an a projective algebraic surface S. Let g be the genus of a smooth curve in the linear system $|L|$. If L is sufficiently ample with respect to δ, the number of $N^L_{g-\delta}$ of δ-nodal curves in a general δ-dimensional sublinear system of $|L|$ will be finite. Kool-Shende-Thomas use the generating function of the Euler numbers of the relative Hilbert schemes of points of the universal curve over $|L|$ to define the numbers $N^L_{g-\delta}$ as BPS invariants and prove a conjecture of mine about their generating function (proved by Tzeng using different methods).

We use the generating function of the χ_y-genera of these relative Hilbert schemes to define and study refined curve counting invariants, which instead of numbers are now polynomials in y, specializing to the numbers of curves for $y = 1$. If S is a K3 surface we relate these invariants to the Donaldson-Thomas invariants considered by Maulik-Pandharipande-Thomas.

In the case of toric surfaces we find that the refined invariants interpolate between the Gromow-Witten invariants (at $y = 1$) and the Welschinger invariants (which count real curves) at $y = -1$. We also find that refined invariants of toric surfaces can be defined and computed by a Caporaso-Harris type recursion, which specializes (at $y = 1, -1$) to the corresponding recursion for complex curves and the Welschinger invariants.

Period relations for certain automorphic motives

Lucio Guerberoff

In this talk we will describe how to prove certain period relations for automorphic motives. More precisely, we will express the Deligne periods of motives attached to automorphic representations of unitary groups over a CM field as a product of certain quadratic periods, which are defined for any polarized regular motive. We will then explain how the critical values of the corresponding L-function can be expressed in terms of the Petersson norm of holomorphic forms. The Tate conjecture implies that this norm also factorizes as the product of the quadratic periods, thus relating the critical values of the L-function with the Deligne period, as predicted by Deligne's conjecture on special values. We will mention some special cases in which the use of the Tate conjecture can be avoided.

Rationality of Cubic Fourfolds

Joe Harris

For two centuries, the question of the rationality of cubic hypersurfaces has played a pivotal role in algebraic geometry. The discovery of the irrationality of cubic plane curves and the rationality of cubic surfaces, and most recently the proof by Clemens and Griffiths that smooth cubic threefolds are irrational, have all been milestones in the subject.
At issue now is whether smooth cubic fourfolds are rational. While the issue is still very much open, it seems likely that it will illuminate another question: whether rationality is an open condition in families of smooth projective varieties, and whether it's closed (both are unknown in general).
In this talk I'll give an overview of the state of our knowledge about cubic fourfolds, the conjectured answer to the question of rationality and why we might believe it's true.

Applications of the fundamental lemma to number theory

Michael Harris

The *Fundamental Lemma*, proved by Ngô Bao Châu in 2007-2008, is an explicit identity between integrals over conjugacy classes of functions on certain pairs of p-adic groups. It was formulated as a conjecture by Langlands and Shelstad in 1987, with two primary motivations: to stabilize the Arthur-Selberg trace formula, in order to establish certain cases of Langlands' functoriality conjectures, and to determine the representations of Galois groups of number fields arising from the cohomology of Shimura varieties. Thanks to the work of Ngô and related results of Arthur, Kottwitz, Waldspurger, Laumon, and many others, a great deal of progress has been made on both of these questions. The lecture will describe the Galois representations that have been constructed with the help of the Fundamental Lemma and will show how knowledge of their existence, combined with the methods introduced by Wiles, can be applied to solve traditional problems in algebraic number theory, such as the Sato-Tate conjecture and the Main Conjectures in Iwasawa theory. If time permits, I will also explain how the fundamental lemma arises in the theory of the stable trace formula.

Noether's theorem

Robin Hartshorne

A theorem attributed to Max Noether states that a general surface in P^3 contains only complete intersection curves. I will say something about the the history of this result, and then outline a proof using deformation theory, based on results described in my lectures the previous week.

On a complex-symplectic mirror pair

Reimundo Heluani

To any Calabi-Yau manifold M one can attach a vertex algebra with two commuting $N = 2$ superconformal supersymmetries, one of them is associated to the complex structure and the other to the symplectic structure. A naive version of Mirror symmetry would require the existence of another Calabi-Yau manifold N and an isomorphism of these vertex algebras exchanging the roles of the complex and symplectic structures.
In the generalized Calabi-Yau case this can be further simplified since we may consider a symplectic manifold M (hence generalized Calabi-Yau) with no compatible complex structure and a complex manifold N endowed with a holomorphic gerbe making N into a (twisted) generalized Calabi-Yau manifold with no compatible Kähler structure. We discuss an example when one can attach to M and N isomorphic algebras intertwining the $N = 2$ supersymmetric structures.

Moduli spaces of framed instanton bundles on $\mathbb{CP}^3$

Marcos Jardim

I will present the ADHM construction of framed instanton sheaves on $\mathbb{CP}^3$, which yields a parametrization of the (fine) moduli space of framed instanton bundles in terms of matrices satisfying some quadratic equations. Using twistor theory, one can show that such moduli space coincides with the space of twistor section of the moduli space of framed bundles on $\mathbb{CP}^2$.
We also introduce a the notion of trisymplectic structures on a complex manifolds, and the notion of trisymplectic reduction. We show that the moduli space of framed instanton bundles can also be described in terms of a trisymplectic reduction, and conclude that it is a smooth trisymplectic manifold of the expected dimension.
Joint work with Misha Verbitsky.

On the solution set of a sparse polynomial equation system

Gabriela Jeronimo

We will focus on recent symbolic algorithms to solve systems of *sparse* multivariate polynomial equations, namely, systems of equations given by polynomials with nonzero coefficients only at prescribed sets of monomials.
The structure of the prescribed monomial sets, the so-called family of *supports* of the system, is closely related to geometric properties of the set of its complex solutions. A fundamental result in sparse elimination is Bernstein's theorem (1975), which states that the number of isolated roots in $(\mathbb{C} \setminus \{0\})^n$ of a generic system of n equations in n variables with given supports is the mixed volume of the family of supports.
In this talk we will present a symbolic probabilistic algorithm to compute all the isolated roots in $\mathbb{C}^n$ of an arbitrary sparse polynomial system and an upper bound for the number of these roots. In addition, we will show combinatorial conditions on the system supports

that enable us to describe algorithmically the equidimensional components of positive dimension of *generic* sparse polynomial systems within good complexity bounds.
This is joint work with María Isabel Herrero and Juan Sabia.

Equations for abelian surfaces with real multiplication

Abhinav Kumar

We outline a method to compute equations for Hilbert modular surfaces using birational maps with moduli spaces of elliptic K3 surfaces. The explicit formulae enable us to determine the Igusa-Clebsch invariants of the corresponding genus 2 curves whose Jacobians have real multiplication by the ring of integers of $K = \mathbb{Q}(\sqrt{d})$, for many values of d. This leads to many examples and families of genus 2 curves over $\mathbb{Q}$ whose Jacobians have extra endomorphisms by $\mathcal{O}_K$, some of which can be paired with tabulated modular forms with q-expansion coefficients in K. This is joint work with Noam Elkies.

Equivariant cohomology of some Springer fibers

Shrawan Kumar

We realize the equivariant cohomology of some Springer fibers (including all the Springer fibers for $SL(N)$) as the affine coordinate ring of an explicitly given affine variety. The proof relies on the Borel homomorphism and the localization theorem. This is a joint work with Claudio Procesi.

Holonomicity of Horn systems

Laura Felicia Matusevich

We characterize which Horn hypergeometric systems are holonomic. To do this, we generalize the process by which a Horn system can be turned into a binomial D-module, a torus-equivariant object, and study the precise relationship between the two, and especially, between their characteristic varieties. This is joint work with Christine Berkesch.

Intersection Space Cohomology and Hypersurface Singularities

Laurentiu Maxim

A recent homotopy-theoretic procedure due to Banagl assigns to a certain singular space a cell complex, its intersection space, whose rational cohomology possesses Poincare duality. This yields a new cohomology theory for singular spaces, which has a richer internal algebraic structure than intersection cohomology (e.g., it has cup products), and which addresses certain questions in type II string theory related to massless D-branes arising during a Calabi-Yau conifold transition. While intersection cohomology is stable under small resolutions, in recent joint work with Markus Banagl we proved that the new theory is often stable under smooth deformations of hypersurface singularities. When this is the case, we showed that the rational cohomology of the intersection space can be endowed with a mixed Hodge structure compatible with Deligne's mixed Hodge structure on the ordinary cohomology of the singular hypersurface.

Symmetries of Varieties

James M[c]Kernan

We describe old and new work on the symmetry group of a variety. In particular we focus on two extreme cases. If the automorphism group is large, it is interesting to try to factor any automorphism into elementary transformations, known as Sarkisov links. If the automorphism group is small, it is interesting to give uniform bounds on the order of the automorphism group.

The tropical Torelli map

Margarida Melo

The classical Torelli map is the modular map from the moduli space of smooth projective curves of genus g to the moduli space of principally polarized Abelian varieties of dimension g, mapping a curve into its Jacobian variety together with its theta divisor. The Torelli theorem asserts that this map is injective and its image has been described in several different ways (the so-called Schottky problem). I will present a joint work in colaboration with Silvia Brannetti and Filippo Viviani in which we establish tropical analogues of these classical results. In particular we construct the moduli space of tropical curves of genus g and the moduli space of Abelian varieties of dimension g and we define the tropical Torelli map. We then study the fibers of this map (tropical Torelli) as a Corollary of a Theorem of Caporaso-Viviani ans we characterize its image (tropical Schottky).

The minimal resolution conjecture for points on a del Pezzo surface. Applications

Rosa M. Miró-Roig

It is a long-standing problem in Algebraic Geometry to determine the Hilbert function of *any* set Z of distinct points on *any* projective variety $X \subset \mathbb{P}^n$. It is well-known that $H_Z(t) \leq min\{H_X(t), |Z|\}$ for any t, and that the equality holds if the points are general. A much more subtle question is to find out the exact shape of the minimal free resolution of I_Z. Mustaţă conjectured that the graded Betti numbers had to be as small as possible (when $X = \mathbb{P}^n$, we recover Lorenzini's conjecture).
In my talk, I will give a brief account of the known results around Mustaţă's conjecture and address it for points on a del Pezzo surface. As an application, I will determine the representation type of any smooth del Pezzo surface (i.e, the behaviour of the associated category of indecomposable ACM bundles).

This is joint work with J.F. Pons-Llopis.

Compact moduli spaces of stable bundles on Kodaira surfaces

Ruxandra Moraru

In this talk, I will examine the geometry of moduli spaces of stable bundles on Kodaira surfaces, which are non-Kaehler compact surfaces that can be realised as torus fibrations over elliptic curves. These moduli spaces are interesting examples of holomorphic symplectic manifolds whose geometry is similar to the geometry of Mukai's moduli spaces on K3 and abelian surfaces. In particular, for certain choices of rank and Chern classes, the moduli spaces are themselves Kodaira surfaces.

Simple vector bundles on $\mathbb{P}^n$ and representatios of quivers

Daniela Moura Prata dos Santos

In this work, we show how to construct simple vector bundles on $\mathbb{P}^n$ with rank n and any homological dimension between 1 and $n-1$. Taking the sheafification of minimal free resolutions of certain algebras, we can also prove, using representations of quivers, that the vector bundles in the short exact sequences assocciated with these resolutions are simple.

The Fano cycle in the Intermediate Jacobian of the cubic threefold

Juan Carlos Naranjo

The Fano surface F of lines in the cubic threefold V is naturally embedded in the intermediate Jacobian $J(V)$, we call "Fano cycle" the difference $F - F^-$, this is homologous to 0 in $J(V)$. We study the normal function on the moduli space which computes the primitive Abel Jacobi class of the Fano cycle. By means of the related infinitesimal invariant we can prove that $F - F^-$ in not algebraically equivalent to zero in $J(V)$. Our study of the infinitesimal variation of Hodge structure for V produces intrinsically a threefold $\Xi(V)$ in $\mathbb{G}$ the Grasmannian of lines in $\mathbb{P}^4$. We show that the infinitesimal invariant at V attached to the normal function gives a section for a natural bundle on $\Xi(V)$ and more specifically that this section vanishes exactly on $\Xi \cap F$, which turns out to be the curve in F parameterizing the "double lines" in the threefold. We prove that this curve reconstructs V and hence we get a Torelli-like result: the infinitesimal invariant for the Fano cycle determines V. This is a joint work with Alberto Collino and Gian Pietro Pirola.

Hecke and Sturm bounds for Hilbert modular surfaces

Ariel Pacetti

If f is a modular form of weight k for a congruence subgroup Γ, it has a Fourier expansion of the form $f(z) = \sum_{n\geq 0} a_n e^{\frac{2\pi i n}{N}}$ for some N (depending on Γ). Hecke studied how many Fourier coefficients determine the form f, in other words, he proved the existence of a constant M such that if $a_n = 0$ for all $0 \leq n \leq M$, then f is the zero form. Sturm studied the same problem "modulo p", i.e. he proved the existence of a constant M such that if

the Fourier coefficients of f are integers and $p \mid a_n$ for all $0 \leq n \leq M$, then $p \mid a_n$ for all n. In both cases, the constant M can be taken to be $\frac{k[SL_2(\mathbb{Z}):\Gamma]}{12}$.
In the talk we will study the same problem for Hilbert modular forms over a real quadratic field. We will define what a Hilbert modular form is, and give an idea of how to prove a similar result (involving some invariants of the surface) implying the vanishing of the form in both situations. This is a work in progress with Jose Ignacio Burgos Gil.

On de Jonquières type birational maps of $\mathbb{P}^3$

Ivan Pan

We consider birational maps $J : \mathbb{P}^n - - > \mathbb{P}^n$ which stabilize (birationally) the set of lines passing through a fixed point; the set of these maps constitutes a subgroup G of the Cremona group Cr_n of $\mathbb{P}^n$. For $n = 2$, generator sets for G and Cr_2 may be given by choosing a single element in G and a subgroup of linear automorphisms. In this talk we first note that for $n = 3$ every generator set for G (or Cr_3) contains uncountable many maps defined by polynomials of degree greater than 1. Second we give generators for G which have a simple geometric structure; it is an open problem to know whether Cr_3 may be generated by elements in G and linear automorphisms.

Nonarchimedean geometry, tropicalization, and metrics on curves

Sam Payne

I will discuss the relationship between the nonarchimedean analytification of an algebraic variety and the tropicalizations of its various embeddings in toric varieties, with attention to the metrics on both sides in the special case of curves. This is joint work with Matt Baker and Joe Rabinoff.

Jumps in the Archimedean Height

Gregory Pearlstein

We answer a question of Richard Hain regarding the asymptotic behavior of the archimedean heights and explain its connection to the Hodge conjecture via the work of Griffiths and Green.

Foliations with numerically trivial canonical bundle

Jorge Vitório Pereira

I will report on a recent joint work with Frank Loray and Frédéric Touzet about the structure of codimension one singular holomorphic foliations with numerically trivial canonical bundle on projective manifolds.

Higher order dual varieties: the toric case

Ragni Piene

Given a projective variety $X \subset \mathbb{P}^m$, the dual variety $X^{(1)} \subset \mathbb{P}^{m\vee}$ is the set of hyperplanes tangent to X. More generally, the kth dual variety $X^{(k)}$ is the set of hyperplanes tangent to X to order k, i.e., containing a kth osculating space to X. A variety is called k-defective if the dimension of $X^{(k)}$ is less than expected. In the case that the embedding $X \subset \mathbb{P}^m$ is toric, with associated polytope P, the degree of $X^{(1)}$ can be expressed in terms of the lattice volumes of the faces of P. We show that there is a similar formula for the degree of the 2-dual of a toric threefold, and we use this expression to show that the only 2-jet spanned toric embedding that is 2-defective is $(\mathbb{P}^3, \mathcal{O}_{\mathbb{P}^3}(2))$, i.e., the toric embedding with associated polytope $2\Delta_3$.
This is joint work with Alicia Dickenstein and Sandra Di Rocco.

Curves on surfaces and Brill-Noether theory

Gian Pietro Pirola

Combing the Brill-Noether theory for curves with the generic vanishing theory on surfaces, we obtain a weak Brill-Noether existence result for line bundles on projective irregular surfaces. We use this result to give a characterization of the symmetric product and of the ordinary product of curves.

Some problems on monomials algebras associated to combinatorial objects

Enrique Reyes

Let $calC = (V, E)$ be a clutter. We denote by $P_{\mathcal{C}}$ and $I_{\mathcal{C}}$ the toric ideal and the monomial ideal associated to $\mathcal{C}$. In this talk we give some conditions about when the monomial ideal $I_{\mathcal{C}}$ is shellable or Cohen-Macaulay. In particular, if $\mathcal{C}$ is a graph, i.e. $calC = G$ we give more specific conditions. In this case, we characterize when the toric ideal is a complete intersection. Furthermore we analyze when the toric ideal P_D is a binomial complete intersection for every D orientation of G.

Polarization of isogeny factors on Jacobians with group actions

A. M. Rojas

We present some of the results developed in [2]. Denote by G a finite group acting on a smooth projective curve Z with total quotient the Riemann sphere. The action of G on Z induces a representation of the group algebra $Q[G]$ of G to the rational endomorphisms of the Jacobian JZ of Z (see [1]). The central idempotents of $Q[G]$ determine subvarieties of JZ, that we called "isogeny factors" because they decompose JZ up to isogeny. We outline a method to compute the type of the induced polarization of the isogeny factors (as abelian subvarieties of JZ).

This is a joint work with Herbert Lange.

References

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Killing-Yano tensors in G structures and in GR

Osvaldo Pablo Santillan

We will describe the presence of Killing-Yano tensors in U(n), SU(n), G2 y Spin(7) structures. In addition the rol played by these tensors in General Relatvity will be pointed out explicitely. In particular, certain important theorems relating the algebraic nature of the curvature tensor of a given space time with the presence of conformal and principal Killing-Yano tensors will be reviewed. Several applications of these results will be briefly discussed.

Topology of moduli spaces of vector bundles on a real algebraic curve

Florent Schaffhauser

Moduli spaces of real and quaternionic vector bundles on a curve can be expressed as Lagrangian quotients and embedded into the symplectic quotient corresponding to the moduli variety of holomorphic vector bundles of fixed rank and degree on a smooth complex projective curve. From the algebraic point of view, these Lagrangian quotients are irreducible sets of real points inside a complex moduli variety endowed with an anti-holomorphic involution. This presentation as a quotient enables us to generalise the equivariant methods of Atiyah and Bott to a setting with involutions, and compute the mod 2 Poincaré series of these real algebraic varieties.

This is joint work with Chiu-Chu Melissa Liu from Columbia University.

Spectral data for principal Higgs bundles

María Laura Schaposnik

In this talk I shall present some ongoing work on principal G-Higgs bundles. In particular, we consider $U(p,q)$ and $SU(p,q)$ and by means of the spectral data associated to the corresponding Higgs bundles we shall give a new description of the associated moduli space. As an application of our method, we shall count the connected components of these moduli spaces.

On the Chern classes of singular varieties

José Seade

Chern classes of complex manifolds have played for decades a major role in complex geometry and topology. There are several extensions of this important concept for singular varieties, each having its own interest and characteristics. Perhaps the most interesting of these are the Schwartz-MacPherson and the Fulton-Johnson classes. The difference amongst these are called Milnor classes. In this talk I will explain all these classes and describe my recent article with Michelle Morgado and Roberto Callejas-Bedregal for compact hypersurfaces in complex manifolds, establishing a deep link between Milnor classes and their Lê cycles. This implies that the Milnor classes determine the topology of the local Milnor fibers at each point of the hypersurface, and converesely: the geometry of the local Milnor fibers determines the Milnor classes.

Chemical reaction systems with toric steady states

Anne Shiu

In a chemical reaction system, the concentrations of chemical species evolve in time, governed by the polynomial differential equations of mass-action kinetics. In general, establishing the existence of (multiple) steady states is challenging, as it requires the solution of a large system of polynomials with unknown coefficients. If, however, the steady state ideal of the system is a binomial ideal, then we show that these questions can be answered easily. This talk focuses on systems with this property, are we say such systems have *toric steady states*. Our main result gives sufficient conditions for a chemical reaction system to admit toric steady states. Furthermore, we analyze the capacity of such a system to exhibit multiple steady states. An important application concerns the biochemical reaction networks networks that describe the multisite phosphorylation of a protein by a kinase/phosphatase pair in a sequential and distributive mechanism.
This is joint work with Carsten Conradi, Mercedes Pérez Millán, and Alicia Dickenstein.

The quest for irreducible homogeneous free divisors

Aron Simis

The notion of a free divisor was introduced by Saito in the complex analytic context. It has ever since shown to be very useful and natural in the theory of hyperplane arrangements and singularity theory. However, families of examples of free divisors in the realm of projective algebraic geometry have not been sufficiently pursued to our view. We consider the problem of spotting families of irreducible homogeneous free divisors over a field of characteristic zero (or high enough/sufficiently constrained characteristic). We revisit a few known examples and add a couple of new ones.

Differential Groups and the Gamma Function

Michael F Singer

I will describe a Galois theory of linear *difference* equations where the Galois group are linear *differential* groups that is, groups of matrices whose entries satisfy a fixed set of polynomial differential equations. These groups measure the differential dependence among solutions of linear difference equations. I will show how this theory can be used to reprove Hoelder's Theorem that the Gamma function satisfies no differential polynomial equation as well as new results concerning differential dependence of solutions of higher order difference equations.

Equivariant cohomology of some Springer fibers

Shrawan Kumar

We realize the equivariant cohomology of some Springer fibers (including all the Springer fibers for $SL(N)$) as the affine coordinate ring of an explicitly given affine variety. The proof relies on the Borel homomorphism and the localization theorem. This is a joint work with Claudio Procesi.

Algebraic integrability of differential r-forms

Márcio G. Soares

The theory of integrability constructed by G. Darboux provides a relation between the integrability of polynomial vector fields and the number of invariant algebraic hypersurfaces they admit. This theory was pursued by H. Poincaré and has been successfully applied to the study of some physical models.

An improvement to Darboux's theory was given by J.P. Jouanolou, in which he characterized the existence of rational first integrals for polynomial 1-forms on $\mathbb{K}^n$ and $\mathbb{P}^n_{\mathbb{K}}$, with $\mathbb{K}$ an algebraically closed field of characteristic zero. Jouanolou's result states that a polynomial 1-form admits a rational first integral if and only if it possesses an infinite number of invariant irreducible hypersurfaces. Moreover, if the number of invariant irreducible hypersurfaces is finite, then it is bounded by

$$\binom{d-1+n}{n} \cdot \binom{n}{2} + 2,$$

where d is the degree of the corresponding Pfaff equation.

E. Ghys extended Jouanolou's result to Pfaff equations on a compact complex manifold. M. Brunella and M. Nicolau proved that Darboux-Jouanolou's theorem holds for Pfaff equations in positive characteristic and for non-singular codimension one transversally holomorphic foliations on compact manifolds. A very nice discrete dynamical version of Jouanolou's theorem was proved by S. Cantat.

Our talk will focus on the following algebraic integrability result:

Theorem.

Let $\mathbb{K}$ be an algebraically closed field of characteristic zero and ω a polynomial r-form of degree d on $\mathbb{K}^n$. If ω admits

$$\binom{d-1+n}{n} \cdot \binom{n}{r+1} + r + 1$$

invariant irreducible algebraic hypersurfaces, then ω admits a rational first integral.
This is joint work with Maurício Corrêa Jr. and Luis Guillermo Martinez Maza.

Hyperelliptic covers and anticanonical rational surfaces

Armando Treibich

Let (X, q) be an ellipitic curve marked at its origin and defined over an algebraically closed field $\mathbf{K}$, of characteristic $\mathbf{p} \neq 2$. We consider finite separable marked covers $\pi : (\Gamma, p) \to (X, q)$, where Γ is a hyperelliptic curve, marked at a smooth Weierstrass point p. Starting with the (so-called) Abel rational map, $(\Gamma, p) \to (Jac\,\Gamma, 0)$, which embeds the open subset of smooth points of Γ into the (generalized) Jacobian of Γ, we construct a group homomorphism $\iota_\pi : (X, q) \to (Jac\,\Gamma, 0)$. We then identify X and Γ with their images and call $\pi : (\Gamma, p) \to (X, q)$ a *hyperelliptic d-osculating cover*, if d is the *osculating order* of X at $0 \in Jac\,\Gamma$ with respect to Γ. In zero characteristic ($\mathbf{p} = 0$) they give all spectral curves associated to solutions of the KdV hierarchy, doubly periodic with respect to the d-th variable.
We study the relations between n, g and d, the degree, the arithmetic genus and the *osculating order* of any such cover. We prove, for any $\mathbf{p} \neq 2$, the inequality $(2g+1)^2 \leq 8(2d\text{-}1)(n\text{-}1) + 9$, as well as $2g + 1 \leq \mathbf{p}(2d\text{-}1)$ whenever $\mathbf{p} > 2$. Moreover, we construct, for any $n, d \in \mathbb{N}^*$, $(d\text{-}1)$-dimensional families of such covers. The latter are obtained through the study of the nef cone of a particular anticanonical rational surfaces, once we complete the list of its curves with negative self-intersection.

Group actions on dg-manifolds and their relation to equivariant cohomology

Bernardo Uribe

Let G be a Lie group acting by diffeomorphisms on a manifold M and consider the image of $T[1]G$ and $T[1]M$ of G and M respectively in the category of differential graded manifolds. We show that the obstruction to lift the action of $T[1]G$ on $T[1]M$ to an action on a $R[n]$-bundle over $T[1]M$ is measured by the G equivariant cohomology of M. We explicitly calculate the differential graded Lie algebra (dgla) of the symmetries of the $R[n]$-bundle over $T[1]M$ and we use this dgla to understand which actions are hamiltonian.

Flipping surfaces

Giancarlo Urzúa

This is part of a joint project with Paul Hacking and Jenia Tevelev, which aims to describe the Kollár–Shepherd-Barron compactification of the moduli space of simply connected $p_g = 0$ surfaces of general type, building on pioneering work of Yongnam Lee and Jongil Park. I will explain a general way to identify stable limits of these surfaces using certain explicit threefold flips. As an application, we prove existence of new simply connected Godeaux surfaces and Dolgachev surfaces $X_{2,3}$ in the boundary of these moduli spaces. For example, in the case $K^2 = 4$, there is a one dimensional boundary component parametrizing Dolgachev surfaces $X_{2,3}$ containing smooth rational curves with self-intersection (-4) and (-5). In this talk, I expect to go through this example in detail.

Symplectic Enumeration

Israel Vainsencher

Given the twisted cubic (t, t^2, t^3), there exists a classical correspondence which associates to each point P in 3–space the plane containing the three points on the curve whose osculating planes pass through P. This turns out to be a synthetic description of the distribution associated to the null-correlation bundle, defined by the twisted differential form induced from the symplectic form. We consider the question of enumerating curves tangent to this distribution.

On cographic toric face rings

Filippo Viviani

Toric face rings were introduced by Stanley as a simultaneous generalization of simplicial face rings (or Stanley-Reisner rings) and toric rings. In this talk, I will report on a joint work with S. Casalaina-Martin and J. L. Kass, in which we study certain toric face rings associated to graphs. The structure of these rings depends on various combinatorial properties of their associated graphs: the poset of totally cyclic orientations, the cographic arrangement of hyperplanes and the Dirichelet-Voronoi polytope. Our motivation for studying cographic toric face rings is that they show up as local rings of compactified Jacobians of nodal curves.