SCALINGS OF PITCHES IN MUSIC

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Abstract

We investigate correlations among pitches in several songs and pieces of piano music by mapping them to one-dimensional walks. Two kinds of correlations are studied, one is related to the real values of frequencies while they are treated only as different symbols for another. Long-range power law behavior is found in both kinds. The first is more meaningful. The structure of music, such as beat, measure and stanza, are reflected in the change of scaling exponents. Some interesting features are observed. Our results demonstrate the viewpoint that the fundamental principle of music is the balance between repetition and contrast.

The aesthetic sense and perception remains a mystery in scientific level. One may expect that its mechanism should be dependent on the structure, especially a kind of collective effect of information possessed by the artistic or natural beauty and then the sensory processing of this information. The quantitative nature of most elements of music make analysis using methods of statistical physics feasible. $1/f$ noise was discovered in an earlier measurement of loudness fluctuations in music[1]. Motivated by what was done recently on DNA sequences and human writings[2,3,4], we study here correlations among pitches according to musical scores. 25 songs and several pieces of piano music are studied[5].

In equally tempered scale, the octave is divided into 12 semitones. A semitone is the interval between two tones whose frequency ratio is the twelfth root of 2. We consider two kinds of correlations, referred to as "frequency-dependent" (FD) one and "frequency-independent" one respectively. In the study of FD correlation, we map the real values of tones to positions within a one-dimensional walk, with a semitone corresponding to the unit of distance. The least of the duration of tones in the piece of music studied is selected as the duration unit. The rests are neglected. Hence the step of the walk, which corresponds to the difference between adjacent pitches, is not a constant. This walk reflects directly the change in the logarithm of frequency. Consequently, notes on the staff just make up an approximate landscape of this so-called "FD musical walk".

The correlation can be quantified by studying the displacement $\Delta X(t,T)$ of the walker after T steps, which is the difference between the positions $X(t+T)$ and $X(t)$,

$$\Delta X(t,T) \,=\, X(t+T) - X(t). \tag{1}$$

$\Delta X(t,T)$ is just the sum of changes in each step, i.e.,

$$\Delta X(t,T) \,=\, \sum_{i=t}^{T-1} x(i), \tag{2}$$

where

$$x(i) = X(i+1) - X(i) \tag{3}$$

is the change in one step. The mean square fluctuation which quantifies the correlation is

$$F^2_{FD}(T) \,=< [\Delta X(t,T) - < \Delta X(t,T) >_t]^2 >_t, \tag{4}$$

where $< >_t$ means average over different t. It is well known[3] that for scaling signals,

$$F^2_{FD}(T) \,\sim\, T^{\alpha}. \tag{5}$$

The corresponding power spectrum of x and ΔX are respectively $S_x(f) \sim f^{1-\alpha}$ and $S_{\Delta X}(f) \sim f^{-1-\alpha}$. For an uncorrelated random walk, $\alpha = 1$, x exhibits white noise while ΔX exhibits $1/f^2$ noise. As $\alpha \to 0$, $x \to f$ noise and $\Delta X \to 1/f$ noise. As $\alpha \to 2$, $x \to 1/f$ noise and $\Delta X \to 1/f^3$ noise.

In the study of FI correlation, again the least duration of note is selected as the unit. Pitches as well as rests are treated just as different symbols. They are represented by different numbers, which are then transformed to binary numbers. As in the studies on DNA sequences and writings, "0" or "1" is mapped to up ($y(i) = +1$) or down ($y(i) = -1$) in a one-dimensional walk, which can be termed "FI musical walk". The displacement is $\Delta Y(t,T) = \sum_{i=t}^{T-1} y(i)$. Similar to FD correlation, mean square fluctuations $F_{FI}^2(T)$ of ΔY are calculated. For scaling signals,

$$F_{FI}^2(T) \sim T^{\gamma}. \tag{6}$$

As a typical example, Fig. 1(a) shows the fluctuation in FD musical walk for the national hymn of France, "Marseillaise". One may observe the periodicity due to strophic form with period $\approx$ 400. We may only consider one, e.g., the first period (stanza). It is obvious that there are three scaling regions, after the first with $\alpha_1 \approx 1.029$ and the second with $\alpha_2 \approx 0.682$, there is a platform region with $\alpha \approx 0$. There is unavoidable oscillation and deviation in the platform region, as well as break down due to finite-size in large enough duration. This feature is found to be universal within 25 famous songs. Usually there are two to three regions with nonzero α's before the final platform.

Quantitative results are summarized in Table 1. From the time and duration unit one may know how long a beat or a measure is. For instance, the unit of "marseillaise" is sixteenth, since the time is 4/4, the duration of a beat is 4 while that of a measure is 16. Hence we see that the region I and region II correspond to a beat and a measure respectively. This is valid for most, though not all, of the songs studied. For "Serenade" by Schubert and "God Save The Queen", it can be thought that regions I and II merge to one. For most of the songs, α_1 is very close to 1, indicating that *usually the pitches within one beat are nearly random*. That α_2 and α_3 are smaller than 1 indicates *non-trivial correlations within duration around a measure*, the power spectrum are between $1/f$ and $1/f^2$. For longer duration, $\alpha \approx 0$, indicating $1/f$ noise. Therefore one may say that the longer the duration, the nearer to $1/f$ noise.

We may make some other observations. The comparatively larger deviation from unity in α_1 for three songs "Auld lang Syne", "My old Kentucky home" and "Alamuhan" show correlations even within one beat in a few songs. It is interesting to note that the platform begins immediately after one beat in the two "Cradle song"s and in "Santa Lucia", which gives similar impression to cradle songs. There are also several other songs for which platform begin in duration shorter than one measure, while for some longer. We also notice on the plots for the two "Serenade"s that the oscillation around the platform region particularly small. For several songs, such as "Away in a Manger", "God Rest You Merry, Gentleman","Auld Lang Syne", "Old Black Joe", "Changing Partner", "Kangding Love Song" and "Alamuhan", α's before the platform are comparatively close.

Fig. 1(b) shows fluctuation in FI musical walk for "Marseillaise". After a fairly flat non-power-law region, which is of little interest since it corresponds to one or two duration unit, there is a region with non-zero exponent γ, then crossover to a platform region with $\gamma \approx 0$. Periodicity due to strophic form is also indicated. The structure indicationg more than one scaling region before the platform in the FD correlation has not been observed in FI correlation. This feature is universal for all the songs. The values of γ's are listed in the last column of Table 1. All γ's are larger than 1. It should be noticed that FD fluctuation is for the move of pitch, while FI fluctuation is for the *summation* of binary representation of pitches. If γ were unity, the pitches would be white noise. $1 < \gamma < 2$ indicates that the pitches are between white noise and $1/f$ noise. $\gamma \approx 0$ indicates that the pitches are nearly f noise while their summations are $1/f$ noise. Therefore, long range correlations also exhibit in FI correlation.

The same schemes are also used to treat several pieces of piano music, both melody and accompaniment are analyzed.

"Courante"s and "Sarabande"s in 4th "English Suite", 5th and 6th "French Suite"s by J. S. Bach and "Variation (G major)" by Beethoven are studied. The general feature is similar to that for the songs, apart from that there is oscillation modulation before the final platform region in the FD fluctuation of "Courante"s in "French Suite"s and in Var. VI in "Variation", as shown in Fig. 2. The quantitative results are summarized in Table 2. We may observe comparative similarities between the two in the binary form, between melody and accompaniment, between "Courante"s or "Sarabande"s in different French Suites, as well as between different parts of one Suite. For FD correlation, it can be observed that the platform begins earlier than songs. For every piece in "English Suite", it begins just after one beat. For most of the others, it begins in duration not longer than one measure. Apart from three, "γ"s are also larger than unity.

Given the scalings, we may renormalize the music by selecting out pitches with a certain distance of duration while neglecting others. As an example, the FD correlation of renormalized "Marseillaise" is shown in Fig. 1(c). It is clear that the scaling behavior remains unchanged while the turning or crossover durations at which scaling changes are reduced by the scaling factor. It is interesting to have renormalized music performed and compared with the original one.

The scaling behavior uncovered here demonstrates quantitatively the viewpoint that the fundamental principle of music is the balance between repetition and contrast[6]. Since the pitch corresponds to logarithm of frequency, our results suggest that it is the logarithm of frequency that matters in processing musical information in human brain. This conjecture is supported by the finding[7] that the smallest perceptible frequency change increases as the frequency change. This can be understood as that the perceptible change in logarithm of frequency is constant.

In closing this article, we would like to point out that many of the other elements of music, such as chord, rhythm, timbre and dynamics etc., have not been taken in account here. Further studies are anticipated concerning both music itself and information processing on it.

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Figure captions:

Fig. 1 Double logarithmic plots of mean square fluctuations as a function of the duration T for the song “Marseillaise”. (a) Frequency-dependent correlation. (b) Frequency-independent correlation. (c) Frequency-dependent correlation for this song renormalized by factor 2.

Fig. 2 Mean square fluctuation of frequency-dependent musical walk for the melody of theme B of the “Courante” in “5th French Suite”. This kind of oscillation modulation structure before the platform is found in several piano pieces other than the most with simple power law scalings.

Table 1 Scaling regions and exponents of mean square fluctuations of musical walks for 25 songs. For each song, author or origin, key and time are indicated in the following the title. Keys are all major, unit is that of duration. Reg. I, II and III gives the different scaling regions before the final platform region for the frequency-dependent correlation, α_1, α_2 and α_3 are the corresponding scaling exponents. γ is the scaling exponent for the frequency-independent correlation.

song	unit	reg. I	α_1	reg. II	α_2	reg. III	α_3	γ
National Hymn of PRC (E. Nie, G, 2/4)	1/16	1∼ 4	0.936	5∼ 8	0.623			1.199
God Save The Queen (UK, A, 3/4)	1/8	1∼6	1.103			7∼17	0.987	1.656
Marseillaise (R. de L'isle, G, 4/4)	1/16	1∼4	1.029	4∼16	0.682			1.567
Star-spangled Banner (J.S. Smith, ^{b}B, 3/4)	1/16	1∼4	1.019	4∼8	0.835	8∼18	0.638	1.584
Silent Night (F. Gruber, C, 3/4)	1/8	1∼3	1.083	4∼10	0.665			1.724
Internationale (French, ^{b}B, 4/4)	1/16	1∼4	0.973	4∼8	0.647	9∼12	0.533	1.548
O,Come,All Ye Faithful (J.F.Wade, A, 4/4)	1/8	1∼2	1.013	2∼6	0.620	7∼12	0.439	1.517
Joy to The World (G.F.Handel, D, 2/4)	1/16	1∼4	1.044	4∼8	0.766	8∼19	0.424	1.561
Away in a Manger (J.R.Murray, F, 3/4)	1/8	1∼2	1.107	2∼4	0.929	5∼12	0.925	1.667
Deck The Hall (Old Welsh Air, F, 4/4)	1/8	1∼2	1.088	3∼7	0.705			1.144
God Rest You Merry, Gentleman (English, G, 4/4)	1/8	1∼2	0.999	3∼8	0.945			1.156
Cherry Ripe (English folk song, ^{b}E, 4/4)	1/16	1∼3	0.969	4∼16	0.588			1.200
Auld Lang Syne (Scottish, F, 2/4)	1/16	1∼3	0.863	4∼8	0.834			1.239
When You and I Were Young, Maggie (American, F, 4/4)	1/16	1∼4	1.026	5∼8	0.703			1.564
Santa Lucia (Neapolitan, C, 3/8)	1/32	1∼2	0.983	2∼4	0.640			1.566
Old Black Joe (S. Foster, D, 4/4)	1/16	1∼8	0.937	9∼12	0.824			1.608
My Old Kentucky Home (S. Foster, G, 4/4)	1/16	1∼4	0.874	4∼8	0.462			1.444
Cradle Song(J. Brahms, F, 3/4)	1/8	1∼2	0.993					1.075
Cradle Song (F. Schubert, ^{b}A, 4/4)	1/16	1∼3	1.031					1.368
Serenade (F. Schubert, F, 3/4)	1/48	1∼18	0.981			19∼25	0.598	1.703
Serenade (C. Gounod, F, 6/8)	1/48	1∼6	0.992	7∼36	0.429			1.511
Changing Partner (American, A, 3/4)	1/8	1∼6	0.974	7∼11	0.959			1.268
Feelings (M. Albert, G, 4/4)	1/48	1∼12	0.979	13∼17	0.686	18∼40	0.438	1.811
Kangding Love Song (folk song in Xikang of China, F, 4/4)	1/8	1∼4	0.967	5∼7	0.883			1.519
Alamuhan (folk song in Uygur of China, F, 4/4)	1/16	1∼4	0.757	5∼8	0.689			1.314

Table 2 Scaling regions and exponents of mean square fluctuations of musical walks for several pieces of piano music. All keys are major. For "Courante" and "Sarabande" of a same Suite, keys are the same hence indicated after the the author Bach, times are different hence indicated separately for "Courant" and "Sarabande". Each piece in Suites is of binary form with two parts A and B. There is oscillation modulation before the platform region in the frequency-dependent correlations of "Courante"s in "French Suites", hence no quantitative analysis is made for them. All parts of "Variation" have the same key and time.

piano music	unit	reg. I	α_1	reg. II	α_2	reg. III	α_3	γ
4th English Suite (J. S. Bach, F)								
Courante(3/2)								
melody A	1/32	1∼4	0.964	5∼8	0.883	9∼16	0.595	0.959
melody B	1/32	1∼4	1.005	5∼8	0.805	9∼16	0.703	1.507
accompaniment A	1/32	1∼4	1.001	5∼8	0.893	9∼16	0.269	1.141
accompaniment B	1/32	1∼4	1.002	5∼8	0.964	9∼16	0.425	1.201
Sarabande(3/4)								
melody A	1/32	1∼4	0.911	5∼8	0.826	9∼16	0.925	1.083
melody B	1/32	1∼4	0.958	5∼8	0.755	9∼16	0.643	1.572
accompaniment A	1/32	1∼16	1.012					1.273
accompaniment B	1/32	1∼8	1.016	9∼16	0.715			1.547
5th French Suit (J. S. Bach,G)								
Courante								
melody A								1.487
melody B								1.144
accompaniment A								1.582
accompaniment B								1.600
Sarabande(3/4)								
melody A	1/32	1∼4	0.938	5∼8	0.839	9∼12	0.632	1.589
melody B	1/32	1∼5	0.914	6∼22	0.675			1.581
accompaniment A	1/16	1∼4	0.972	4∼6	0.708	6∼12	0.411	1.206
accompaniment B	1/16	1∼4	0.950	4∼6	0.526	6∼8	0.352	1.099

Table 2 (Continued)

piano music	unit	reg. I	α_1	reg. II	α_2	reg. III	α_3	γ
6th French Suite (J. S. Bach, E)								
Courante()								
melody A								1.292
melody B								1.588
accompaniment A								1.676
accompaniment B								1.332
Sarabande(3/4)								
melody A	1/16	1∼8	0.769	9∼12	0.471			1.668
melody B	1/16	1∼7	0.848	8∼12	0.676			1.620
accompaniment A	1/32	1∼8	1.024					1.693
accompaniment B	1/32	1∼8	1.043	9∼32	0.908			1.687
Variation (L. v. Beethoven, G, 3/4)								
melody								
Theme	1/32	1∼4	1.005	4∼8	0.666			0.854
Var. I	1/32	1∼2	1.003	3∼6	0.918			1.338
Var. II	1/32	1∼4	1.001	4∼8	0.308			1.219
Var. III	1/32	1∼4	0.946	4∼6	0.615			1.263
Var. IV	1/32	1∼4	1.004	4∼8	0.563			1.112
Var. V	1/96	1∼16	0.997	17∼40	0.800			1.503
accompaniment								
Theme	1/32	1∼4	1.004	4∼8	0.273			0.822
Var. I	1/32	1∼4	1.005	4∼8	0.155			1.118
Var. II	1/32	1∼2	1.003	3∼19	0.938			1.049
Var. III	1/32	1∼4	1.014	4∼6	0.722			1.295
Var. IV	1/32	1∼4	1.015	4∼8	0.737			1.355
Var. V	1/32	1∼8	1.021	9∼20	0.931			1.143