Computational approach to multifractal music

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Abstract

In this work we perform a fractal analysis of 160 pieces of music belonging to six different genres. We show that the majority of the pieces reveal characteristics that allow us to classify them as physical processes called the $1/f$ (pink) noise. However, this is not true for classical music represented here by Frederic Chopin's works and for some jazz pieces that are much more correlated than the pink noise. We also perform a multifractal (MFDFA) analysis of these music pieces. We show that all the pieces reveal multifractal properties. The richest multifractal structures are observed for pop and rock music. Also the viariably of multifractal features is best visible for popular music genres. This can suggest that, from the multifractal perspective, classical and jazz music is much more uniform than pieces of the most popular genres of music.

Keywords: Fractal, Fractal dimension, Mulifractality, Singularity spectrum.

1. Introduction

Since B. Mandelbrot's "Fractal Geometry of Nature" was published (Mandeldrot, 1982), fractals have an enormous impact on our perception of the surrounding world. In fact, fractal (i.e. self-similar) structures are ubiquitous in nature, and the fractal theory itself contitutes a platform on which various fields of science, such as biology (Ivanov et al., 1999; Makowiec et al., 2009;

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Rosas et al., 2002), chemistry (Stanley and Meakin, 1988; Udovichenko and Strizhak, 2002), physics (Muzy et al., 2008; Oświęcimka, 2006; Subramaniam et al., 2008), and economics (Drożdż et al., 2010; Kwapień et al., 2005; Matia, 2003; Oświęcimka et al., 2005; Zhou, 2009), come across. This (statistical) self-similarity concerns irregularly-shaped empirical structures (Latin word fractus means 'rough') which often elude classical methods of data analysis. This interdisciplinary character of the applied fractal geometry is not confined only to science, but also in art, which may be treated as some reflection of reality, some interesting fractal features might be discerned. An example of this are the fractal properties of Jackson Pollock's paintings (Taylor et al., 1999) and the Zipf's law describing literary works (Kwapień et al., 2010; Zanette, 2006; Zipf, 1949). In a course of time the fractal theory encompassed also the multifractal theory dealing with the structures which are convolutions of different fractals. It turned out that such structures and corresponding processes are not rare in nature and the proposed multifractal formalism allowed researchers to introduce distinction between mono- and multifractals (Halsey et al., 1986). Development of those intriguing theories would not have been possible, though, if there had not been substantial progress in computer science. On the one hand, fractals - due to their structure - can easily be modelled by using iterative methods, for which the computers are ideal tools. On the other hand, however, the multifractal analyses require significant computing power. The result of such an analysis is identification of diverse patterns in different subsets of data which would be impossible without modern computers. Although relations of mathematics and physics with music date back to ancient times (Pythagoras of Samos considered music a science of numbers), a new impulse for them arrived together with developments in the fractal methods of time series analysis (Bigerelle and Iost, 2000; Ro and Kwon, 2009). The first fractal analysis of music was carried out in 1970s by Voss and Clarke (Voss and Clarke, 1975), who showed that the frequency characteristics of investigated signals behave similar to $1/f$ noise. Interestingly, this type of noise (called pink noise or scaling noise) occurs very often in nature (Bak et al., 1987). The $1/f$ spectral density is an attribute, among others, of meteorological data series, electronic noise occurs in almost all electronic devices, statistics of DNA sequences and heart beat rthythm (Bak, 1996). Thus, from this point of view, music imitates natural processes. A note worth making here is that, according to the authors of the above-cited article, the most pleasent to ear kind of music is just the pink noise. In 1990s Hsu and Hsu showed that for some classical pieces of Bach

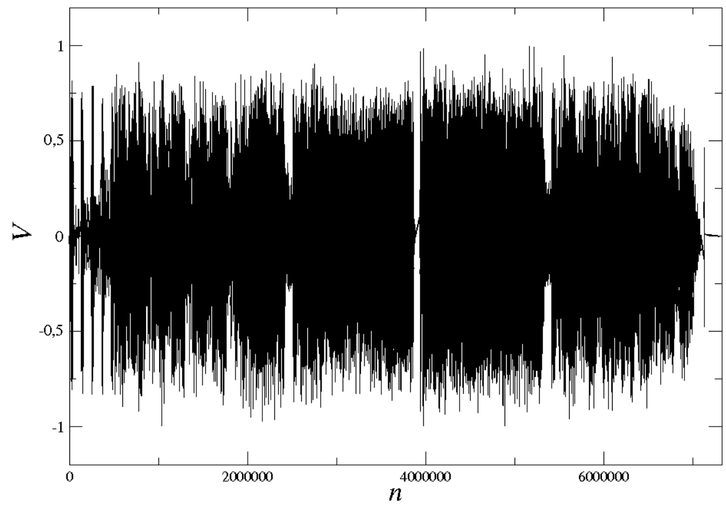

Figure 1: Exemplary time series representing the sound wave of the song "Good Times Bad Times" by Led Zeppelin.

and Mozart and for some children songs, a power law relation occurs between the number of subsequent notes F, distant from each other by i semitones as a function of i (Hsu and Hsu, 1990):

$$F = c/i^D \tag{1}$$

where c denotes a propotionality constant and D is the fractal dimension ($1 < D < 2.25$). In contrast, no similar relation has been observed for some works of Karlheinz Stockhausen, one of modern composers belonging to the strict musical avant-garde. It should be mentioned that the relation discovered by Hsu and Hsu (1990). can be considered as an expression of the Zipf's law in music. In recent years, a more advanced multifractal analysis was carried out (Jafari et al., 2007; Su and Wu, 2006). For example, by substituting both the rhythm and melody by a geometrical sequence of points, Su and Wu (2006) showed that these quantities can be considered the multifractal objects. They also suggested that various genres of music may possess their genre-specific fractal properties. Thus, there might exist a multifractal criterion for classifying a musical piece to a particular genre. Music can be considered a set of tones or sounds ordered in a way which is pleasant to ear. And although the reception of a musical piece is subjective, music affects a listener irrespective of his sensitivity or musical education (Storr, 1997).

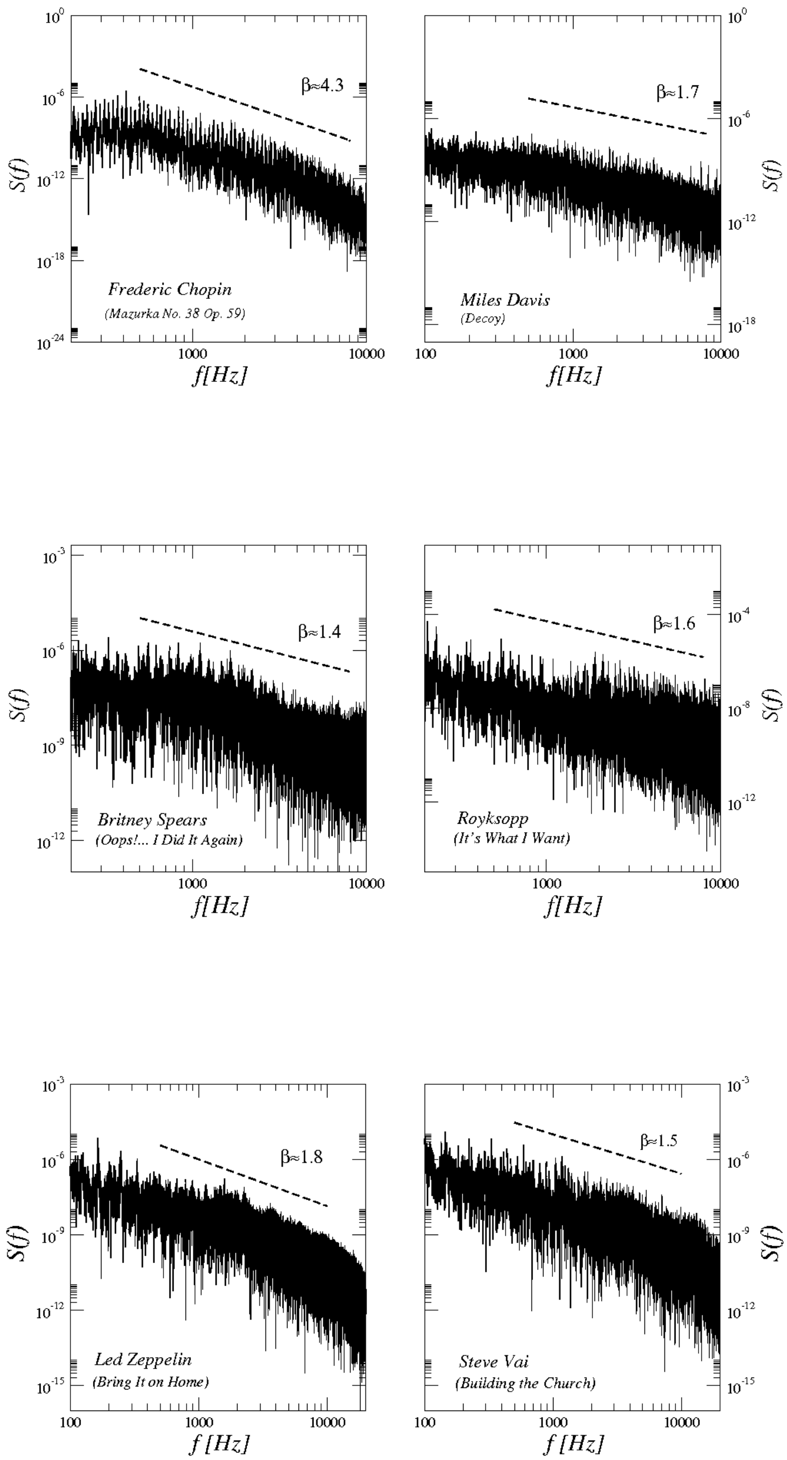

Figure 2: Exemplary power spectra (in log-log scale) for six pieces of music representing six genres (from top to bottom: classical music, jazz, pop music, electronic music, rock, and hard rock). Power-law trends in each panel are indicated by dashed lines, whose slopes correspond to the corresponding values of β.

Therefore, a hypothesis which arises in this context is that music as an object may refer not only to the structure of a musical piece but also to the way it is perceived.

2. Power spectral analysis

In our work we analyzed 160 pieces of music from six popular genres: classical music, pop music, rock, hard rock, jazz, and electronic music. The first one, classical music, was represented by 38 works by Frederick Chopin, divided into three periods of his career. Pop music consisted of 51 songs performed by Britney Spears, rock and hard rock music - 20 songs performed by Led Zeppelin and 20 songs by Steve Vai, respectively, jazz - 25 compositions performed by Miles Davies or Glenn Miller. Finally, an electronic music consisted of 6 pieces of music by Royskopp. All the analyzed pieces were written in the WAV format. In this format the varying amplitude of a sound wave $V(t)$ is encoded by a 16-bit stream sampled with 44,1 kHz frequency. After encoding, the amplitude $V(t)$ was expressed by a time series of length depending on the temporal length of a given piece of music (several million points, on average). An exemplary time series encoding a randomly selected song is displayed in Figure 1. We started our analysis with calculating the power spectrum $S(f)$ for each piece of music. This quantity carries information on power density of sound wave components of frequency $(f; df)$. According to the Wiener-Khinchin theorem, $S(f)$ is equal to the Fourier transform of autocorrelation function or, equivalently, the squared modulus of a signal's Fourier transform:

$$S(f) = |X(f)|^2 \tag{2}$$

where

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-2\pi i f t} dt \tag{3}$$

is the Fourier transform of a signal $x(t)$. If the power spectrum decreases with f as $1/f^{\beta}$, $(\beta \geq 0)$, it means that the signal under study is characterized by log-range autocorrelation within the scales described by the corresponding frequencies f. The faster is the decrease of $S(f)$ (i.e. the higher value of β), the stronger is the autocorrelation of the signal. It is worth recalling here that the Brownian motion corresponds to $\beta = 2$, while the white noise (uncorrelated signal) to $\beta = 0$. Since the exponent β can easily be transformed into the Hurst exponent (a well-known notion in fractal analysis) or

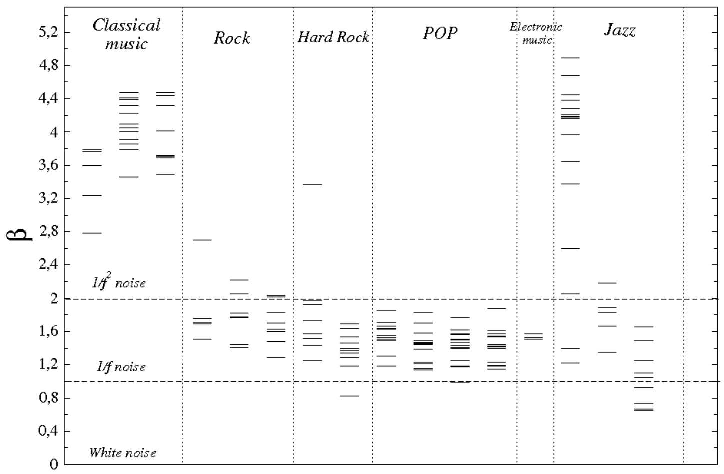

Figure 3: Exponent β calculated for each piece of music analyzed in the present work (short horizontal lines). Columns correspond to individual artists, periods of their career (Chopin), or albums. Dotted vertical lines separate different music genres.

into the fractal dimension, the power spectrum can be classified among the monofractal techniques of data analysis. The power spectra were calculated for each piece of music. In most cases, the graph $S(f)$ was a power-law decreasing function for frequencies 0.1-10 kHz with the slope characteristic for a given piece. The notable exceptions were works of Chopin for which the graphs were scaling between 1 and 10 kHz. Six representative spectra for different genres are shown in Figure 2. To each empirical spectrum $S(f)$ a power function was fitted (the straight lines in Figure 2) within the observed corresponding power-law regime. A slope of the fitted function corresponds to the exponent β. All the calculated values of β, are exhibited in Figure 3. As it can be seen, the highest values of β correspond to works of F. Chopin (classical music,$\beta_{MAX} =\ 4.4$) and some works of Glenn Miller (jazz, $\beta_{MAX} =\ 4.8$). Exponents in these cases are much higher than 2 which means that the underlying processes are more correlated than the Brownian motion. Also several songs by Led Zeppelin (rock) have $\beta > 2$ but not so prominent as the pieces from those genres mentioned before. Interestingly, β for Led Zeppelin declines with time. For their chronologically first album, the highest exponent is 2.8, while for the subsequent albums it drops to 2.3 and 2.1, respectively. For the other analyzed music genres, i.e. electronic, pop, rock and hard rock music, $1 < \beta < 2$. It is also worth mentioning that for several jazz pieces, the exponent β drops below 1, which means that they approach white noise. An author of these songs is Miles Davies, one of the most significant jazz artists, who often was a precursor of new styles and sounds. To summarize this part of our analysis, we can say that from the power spectrum perspective, the majority of the analyzed pieces of music can in fact be considered the $1/f$ processes. This is even more evident for more popular music genres like pop and rock than for rather exclusive genres like jazz and classical music.

3. Multifractal analysis of musical compositions

In order to have a deeper insight into dynamics of the investigated signals we performed also multifractal analysis of data. We used one of the most popular and reliable methods - the Multifractal Detrended Fluctuation Analysis (MFDFA) (Kantelhardt, 2002). This method allows us to calculate fractal dimensions and Hoelder exponents for individual components of a signal decomposed with respect to the size of fluctuations. Consecutive steps of this procedure are presented below. At the beginning we calculate the so-called profile, which is the cumulative sum of the analyzed signal:

$$Y(i) = \sum_{j=1}^{i} [x_j - \langle x \rangle] \quad \text{for} \quad i = 1, 2, ...N, \tag{4}$$

where $\langle x \rangle$ donotes the signal's mean, and N denotes its length. The subtraction of the mean value is not necessary, because a trend is eliminated in the next steps. Then we divide the profile $Y(i)$ into N_s disjoint segments of lentgh s ($N_s = N/s$). In order to take into account all the points (at the end of the signal's profile some data can be neglected), the dividing procedure has to be repeated starting from the end of $Y(i)$. In consequence, we obtain $2N_s$ segments. In each segment ν, the estimated trend is subtracted from the data. The trend is represented by a polynomial P^l_ν of order l. The polynomial's order used in calculation determines the variant of the method. Thus, for $l = 1$ we have MFDFA1, for $l = 2$ - MFDFA2, and so on. After detrending the data, its variance has to be calculated in each segment:

$$F^2(s, \nu) = \frac{1}{s} \sum_{i=1}^{s} \{Y[(\nu - 1)s + i] - P^l_\nu(i)\}^2 \quad \text{for} \quad \nu = 1, 2, ...N_s \tag{5}$$

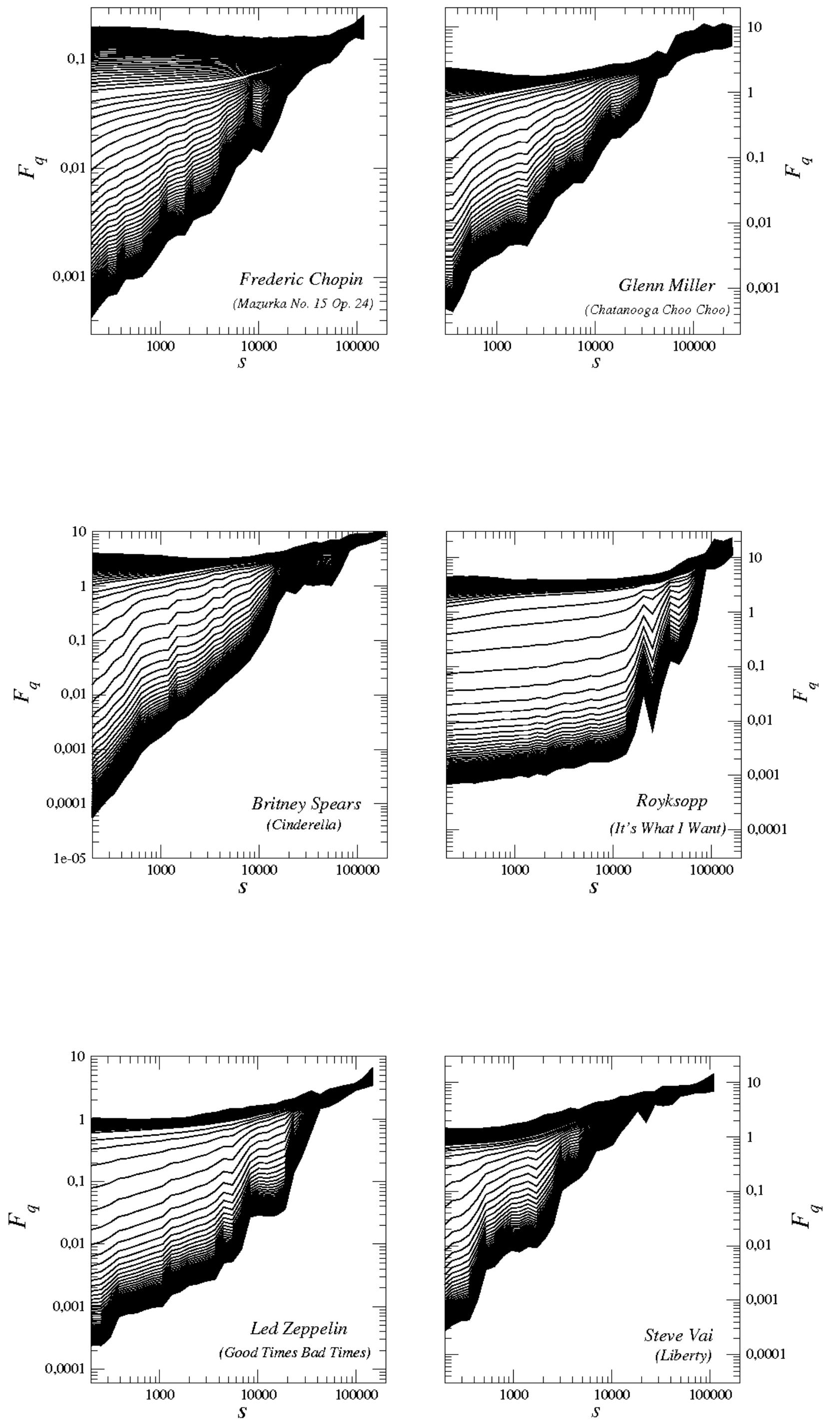

Figure 4: Exemplary fluctuation function Fq (in $log - log$ scale) for six pieces of music representing six genres (from top to bottom: classical music, jazz, pop music, electronic music, rock and hard rock). Each line represents Fq calculated for particular q values in the range from -4 to 4

or

$$F^2(s,\nu) = \frac{1}{s}\sum_{i=1}^{s}\{Y[N-(\nu-N_s)s+i]-P_\nu^l(i)\}^2 \quad \text{for} \quad \nu = N_s+1, N_s+2, ...2N_s \tag{6}$$

The variances are then averaged over all the segments and, finally, one gets the qth order fluctuation function given by:

$$F_q(s) = \left\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}[F^2(s,\nu)]^{q/2}\right\}^{1/q}, \tag{7}$$

where the exponent q belongs to $\mathbb{R}\setminus\{0\}$. This procedure has to be repeated for different values of s. If the analyzed signal has any fractal properties, the fluctuation function behaves as:

$$F_q(s) \sim s^{h(q)}, \tag{8}$$

where $h(q)$ denotes the generalized Hurst exponents. A constant $h(q)$ for all q's means that the studied signal is monofractal and $h(q) = H$ (the ordinary Hurst exponent). For multifractal signals, $h(q)$ is a monotonically decreasing function of q. It can be easily noticed that, by varying the q parameter, it is possible to decompose the time series into fluctuation components of different character: for $q > 0$ the fluctuation function mostly describes large fluctuations, whereas for $q < 0$ the main contribution to the F_q comes from small fluctuations. By knowing the $h(q)$ spectrum for a given set of data, we can calculate its singularity (multifractal) spectrum:

$$\alpha = h(q) + qh^{'}(q) \quad \text{and} \quad f(\alpha) = q[\alpha - h(q)] + 1, \tag{9}$$

where $h^{'}(q)$ stands for the derivative of $h(q)$ with respect to q, the Hoelder exponent α donotes singularity strength, and $f(\alpha)$ is the fractal dimension of the set of points characterized by α. For a monofractal time series, the singularity spectrum reduces to a single point $(H, 1)$, while for multifractal time series, the spectrum assumes the shape of an inverted parabola. The multifractal strength is a quantity which describes the richness of multifractality, i.e., how diverse are values of the Hoelder exponents in a data set. It can be estimated by the width of the $f(\alpha)$ parabola:

$$\Delta\alpha = \alpha_{max} - \alpha_{min} \tag{10}$$

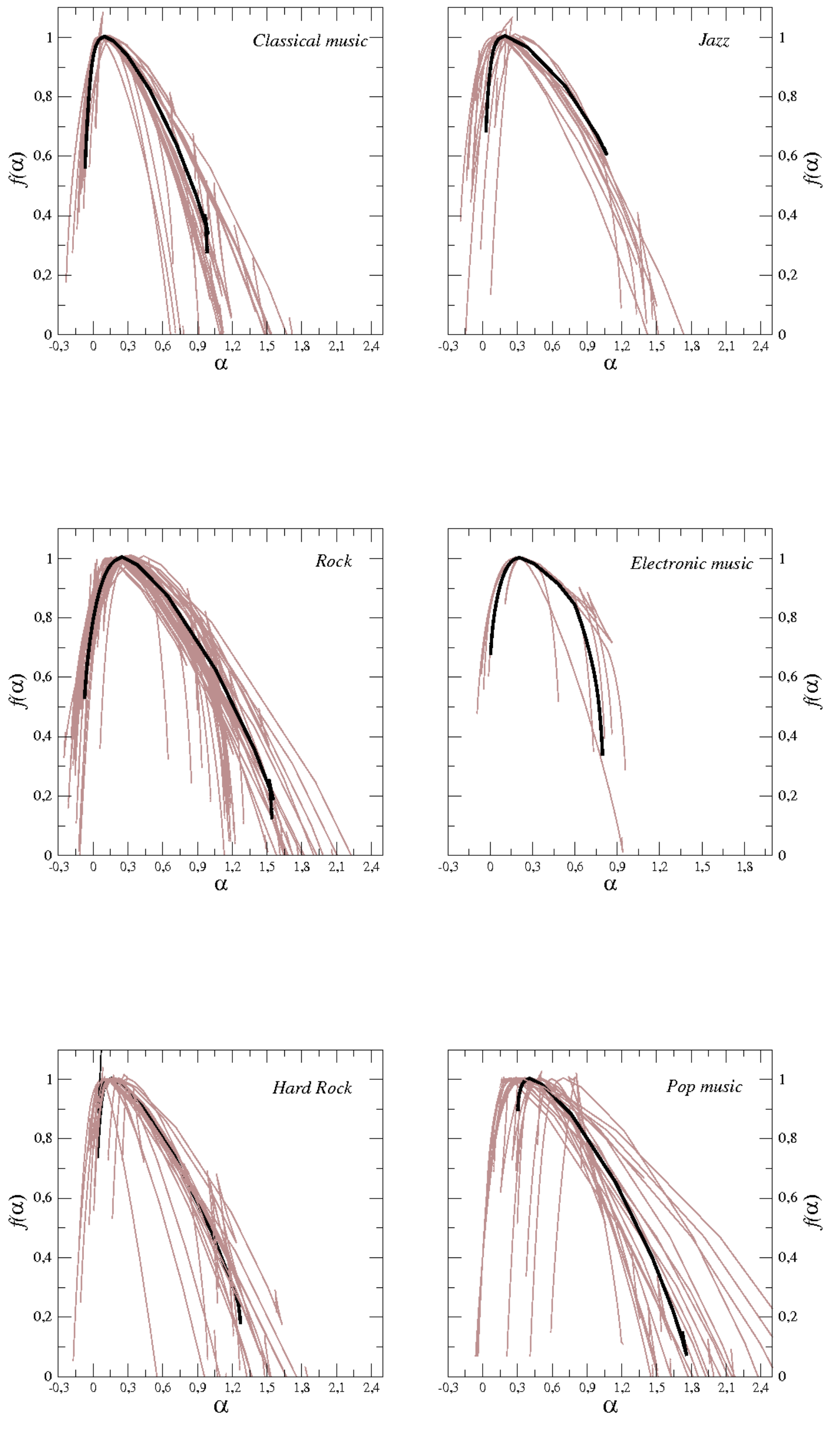

Figure 5: The multifractal spectra $f(\alpha)$ calculated for each analyzed piece of music (brown lines) and its mean spectrum (black lines).

where α_{min} and α_{max} stand for the extreme values of α. The bigger is α the richer is the multifractal.

Using MFDFA2, which guarantees stability of results, we calculated the fluctuation function Fq for all the analyzed signals in the scale s range from 50 to 100,000 points. The value of q was increased by 0.2 in the range from -4 to 4. Exemplary fluctuation functions for our six music genres are shown in Figure 4. All the calculated Fq functions are characterized by a power law dependence on the scale for all q's. However, the range of scaling varies slightly for different pieces. By looking at the shown examples, it is easy to notice that for F. Chopin, Britney Spears, Glenn Miller, and Steve Vai, the scaling involves almost all the considered values of s, while for electronic music we can distinguish two scaling ranges: the longer one for the scales $40 < s < 10,000$ and the shorter one for $10,000 < s < 100,000$. Such double scaling appears also occasionally for the other genres of music. However, for the most cases, we observe only one type of scaling. In Figure 4 we can also notice a clear dependence of the $h(q)$ exponent (the slope coefficient of Fq in double log scale) on q. And so, the largest values of $h(q)$ correspond to $q < 0$, whereas for $q > 0$, $h(q)$ takes smaller values. Therefore already at this stage of calculations, it can be seen that the analyzed signals can have distinct multifractal properties. It is also worth to mention that for the large scales (e.g. for jazz $s > 40,000$, for hard rock $s > 20,000$), scaling loses its multifractal traits, and $h(q)$ does not depend on q. It is related to the limited range of nonlinear correlations (Drożdż et al., 2009). The scale s, for which the scaling character of Fq changes, sets a limit for estimation of the multifractal spectrum. For all the fluctuation functions, we estimated the singularity spectra $f(\alpha)$. Figure 5 presents the multifractal spectra (grey lines) and the corresponding mean multifractal spectra (black lines) for the music genres to which the given pieces belong. All the mean spectra are asymmetric. The right part which describe the small amplitude fluctuations is clearly longer. This effect is best visible for the rock, hard rock, and pop songs. Locations of the extrema of these spectra ($\alpha \approx 0.2$) suggest considerably antipersistent behavior of the analyzed time series. We can easily see that the width of the multifractal spectra for a particular genre fluctuates considerably. Nevert heless, all the spectra are characterized by the widths large enough that they can be regarded as multifractal structures. This confirms observation made above for the Fq function. The narrowest mean multifractal spectrum was observed for electronic music ($\Delta\alpha = 0.85$). Classical music and jazz display mutually comparable widths of, respectively, 1.0 and 1.1. The widest mean

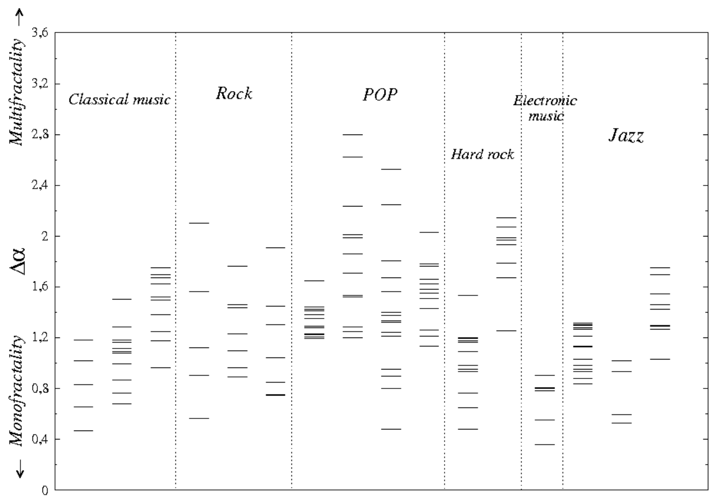

Figure 6: Value of $\Delta\alpha$ calculated for each analyzed piece of music (short horizontal lines). Columns correspond to individual artist, periods of their career, or albums. Dotted vertical lines separate different genres of music.

$f(\alpha)$ is seen for hard rock (1.22), rock (1.5), and pop (1.8). Thus, from this point of view, the richest multifractal (the richest dynamics of processes) is an attribute of the most popular music genres. The more exclusive genres are characterized by poorer multifractals. Figure 6 presents a collection of all the calculated widths of $f(\alpha)$. Vertical lines separate different music genres and each piece is represented by a single horizontal line. As it can be seen, the most variable multifractal spectra widths characterize pop ($0.5 < \alpha < 2.8$), rock ($0.5 < \alpha < 2.1$) and hard rock ($0.51 < \alpha < 2.15$) music. Thus, on account of multifractal properties, the pieces belonging to these genres differ markedly among themselves. Much more consistent from this point of view are the pieces of classical music, jazz and electronic music. We can draw therefore a conclusion that this is the richness of multifractal forms what distinguishes popular music from the more exclusive and the less listened to musical genres.

4. Conclusions

To sum up, our work presents results of a fractal analysis of selected music works belonging to six different genres: pop, rock, hard rock, jazz, classical, and electronic music. The results confirm that the amplitude signals $V(t)$

are characterized by the power spectrum falling off according to a power law: $S(f) \sim 1/f^{\beta}$. Interestingly, rate of this fall can be characteristic for a particular genre. For classical music and some pieces of jazz, $S(f)$ declines the fastest, while for popular music (pop, rock, hard rock , and electronic music) the power spectrum falls more slowly suggesting less correlated signals. The same signals were also subject to a multifractal analysis. It turned out that such data demonstrate well-developed multifractality. Interestingly, the most variable widths of multifractal spectra (and also the widest singularity spectra thus strongest nonlinear correlations) were observed for popular genres like pop and rock. For the remaining genres, the multifractal properties were rather similar among the pieces. Therefore, from this point of view, the popular music is characterized by the amplitude signals with different degree of correlations, whereas more sophisticated musical genres (classical, jazz) are more consistent in this matter.

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