

Fractal String Generation and Its Application in Music Composition

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Abstract

Music is a string of some of the *notes* out of 12 *notes* (Sa, Komal_re, Re, Komal_ga, Ga, Ma, Kari_ma, Pa, Komal_dha, Dha, Komal_ni, Ni) and their *harmonics*. Each note corresponds to a particular *frequency*. When such strings are encoded to form discrete sequences, different frequencies present in the music corresponds to different *amplitude* levels (value) of the discrete sequence. Initially, a class of discrete sequences has been generated using logistic map. All these discrete sequences have at most n-different amplitude levels (value) (depending on the particular raga). Without loss of generality, we have chosen two discrete sequences of two types of Indian raga viz. Bhairabi and Bhupali having same number of amplitude levels to obtain/search close relatives from the class. The relative / closeness can be assured through correlation coefficient. The search is unbiased, random and non-adaptive. The obtained string is that which maximally resembles the given two sequences. The same can be thought of as a music composition of the given two strings. It is to be noted that all these string are fractal string which can be persuaded by fractal dimension.

Keywords: *Bhairabi, Bhupali, Logistic map, Fractal dimension and Correlation Coefficient.*

1. Introduction and Review:

Music is a field of study that has an obvious relationship to mathematics. Music is, to many people, a nonverbal form of communication, which reaches past the human intellect directly into the soul. However, music is not really created by mankind, but only discovered, manipulated and reorganized by mankind. In reality, music is first and foremost a phenomenon of nature, a result of the principles of physics and mathematics. A musical scale is a discrete set of pitches used in making or describing music. Precisely, Music can be encoded as a string of at most 12 notes viz. (Sa, Komal_re, Re, Komal_ga, Ga, Ma, Kari_ma, Pa, Komal_dha, Dha, Komal_ni, Ni) and their harmonics [1, 2, 3, 4]. In this paper, without loss of generality, two different types of Indian raga have been considered. Initially, two music strings of 1000 length have been taken from each of the raga. A class of discrete sequences has been generated through an illustrious chaotic map viz. logistic map. From the class, we have devised a mechanism to find out a close relative to the two those strings. The mechanism is based on correlation coefficient. Before going into the technical details, let us revisit logistic map very briefly.

Logistic Map: The one dimensional, polynomial map of degree 2 is given as

$$l_{n+1} = \lambda \cdot l_n \cdot (1 - l_n) \quad n \geq 0$$

Where l_n belongs to the (0, 1) and l_0 represents the initial value. λ is a positive number, and represents as a control parameter of the chaotic system. The dynamical behaviour depends on entirely on the value of λ [5, 6].

1.1 String generation using Logistic Map:

Using Logistic map a set of sequence of length 1000 have been generated. Then music strings corresponding to different ragas can be generated. The work is carried on with two famous Indian raga viz. *Bhairabi and Bhupali*.

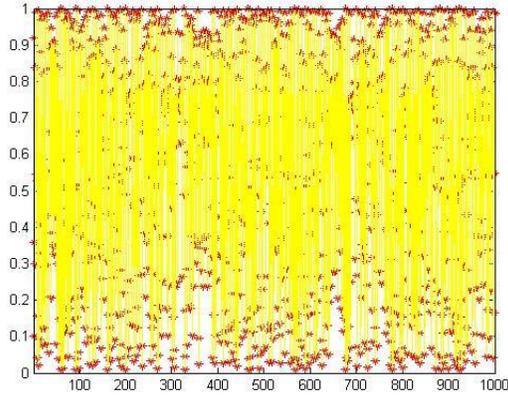
1.1.1 Bhairabi:

In Bhairabi raga, there are seven fundamental frequencies. They are *Sa, Komal_re, Komal_ga, Ma, Pa, Komal_dha, and Komal_ni*. So a music string in Bhairabi raga consists of these seven notations along with the Octet Sa (Sa') and Pause (corresponds to zero frequency). It is evident that in the Logistic map an one dimensional string of number between (0, 1) would come out. Accordingly, Logistic map with 9 frequencies and 1000 length is used to generate various music strings in Bhairabi raga. In the strings *Sa, Komal_re, Komal_ga, Ma, Pa, Komal_dha, Komal_ni*, Octet Sa and Pause are encrypted as S, r, g, M, P, d, n, C and B. The frequencies allotted for different encrypted notes in Bhairabi are given in Table-1.

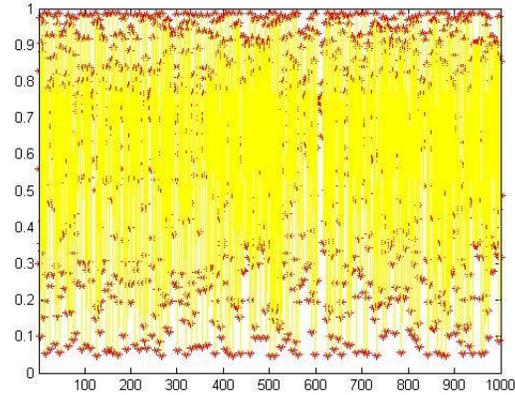
<i>Notes (Encrypted)</i>	<i>Threshold</i>
S	(0, 1/9)
R	(1/9, 2/9)
G	(2/9, 1/3)
M	(1/3, 4/9)
P	(4/9, 5/9)
D	(5/9-2/3)
N	(2/3, 7/9)
C	(7/9, 8/9)
B	(8/9, 1)

Table-1: The frequencies allotted for different encrypted notes in Bhairabi

The graphical representations of the logistic maps corresponding to the given strings are shown:



Bhairabi_1



Bhairabi_2

1.1.2 Bhupali:

In Bhupali raga, there are five fundamental frequencies. They are Sa, Re, Ga, Pa and Dha. So a music string in Bhupali raga consists of these five notations along with the Octet Sa (Sa⁷) and Pause (corresponds to zero frequency). Logistic map with 7 frequencies and 1000 length is used to generate various strings of this particular raga. In the strings Sa, Re, Ga, Pa and Dha, Octet Sa and Pause are encrypted as S, R, G, P, D, C and B. The frequencies allotted for different encrypted notes in Bhupali are given in Table-2.

<i>Notes (Encrypted)</i>	<i>Threshold</i>
S	(0, 1/7)
R	(1/7, 2/7)
G	(2/7, 3/7)
P	(3/7, 4/7)
D	(4/7, 5/7)
C	(5/7-6/7)
B	(6/7, 1)

Table-2. The frequencies allotted for different encrypted notes in Bhupali

Out of them two strings are given below:

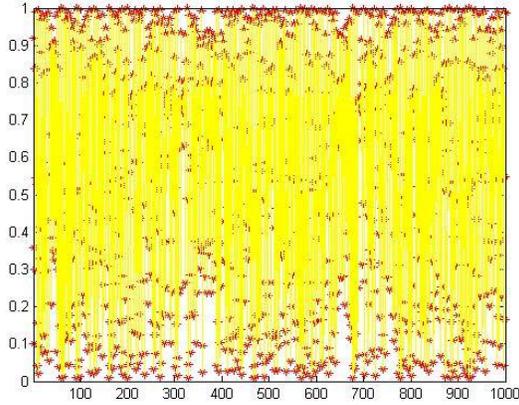
Bhupali_1(corresponding to $\lambda = 3.99$, initial point = 0.1):

SGBGCPBSRPBSSGBGSPBSGCDBSPBSGCPBSRDBGBRDBGBGGBPBSGCPBSRPBSSRPBSSRDBS
GBSPBSRDBSGBRDBGBGRPBSSRPBSSRDBGBRCCDBRDBSPBSGCPBSSGBGPBSSGBPBSSGBSG
BGBGPBSRCCDBRDBGCPBSSRPBSSPB SRCDCDBSGBGRPB SRPBSSRCD BRDCPBSSRPBSRCCC
DCDBSPBSSPB SRDBSPBSRCD BPBSSRCD BGRDBGBGCD BRPBSSPB SRCDBGRPBSSRDBRDBSGB
RCCCCDBRDBSGBRCCDBPBSRPBSSRCCDBRCCCCDBRCD CPBSSRPBSRCD BRDBRDBSGBRCCD
BGBGCPBSSGBGRCD BRCCDBGBGRCCDBRCD BGPBSRPBSSGBRCCDBGPBSSGPBSSGBRPBS
GCPBSSGSPBSRDBPSSPB SRCDBRDBGBSGBGGBSGBRPBSSRPBSSRDBSGBRPB SRCDCPBSS
RDBSPBSRDCPB SRCDCPBSSGSPBSSGBSGBGGBRDBSPBSSGBRCD CPBSRDBGCD BRPBSSGB
RPBSRPBSSRPBSSPBSSGPB SRCDCPBSSRDBRPBSSGBRDBPBSRPBSRPBSRDBRCD CPBSSGPB
SRDCDBSPBSRDBRCCDBRCD BRCD BRDBEGBRDBSGBRPBSSRPBSSPB SRCDBRDBSGBRDBS
SSRCD BPBSRDBGBGPBSGCPBSSRCCDBGRPBSSGPBSSPB SRCDBRCD BRCCDBRDBSGBRDBS
PBSSGCPBSSRPBSRDBGRPBSSPB SRPBSSRCCDBPBSRDBGBGSPBSRDBRPBSRPBSRPBSSGB
RDBGPBSRDBRCCDBGCD BRPB SRCDCPBSSRDBRDCPBSSRPBSSGBRDBGCPBSSRDBSGBSGBR
PBSSPBSSGCPBSSRPBSSRDBGCPBSSPB SRCDBRDCPB SRDCPBSSGBRCCDBRDBGBGBSGBRCCDC
PBSRDBSGBRDCPB SRPB

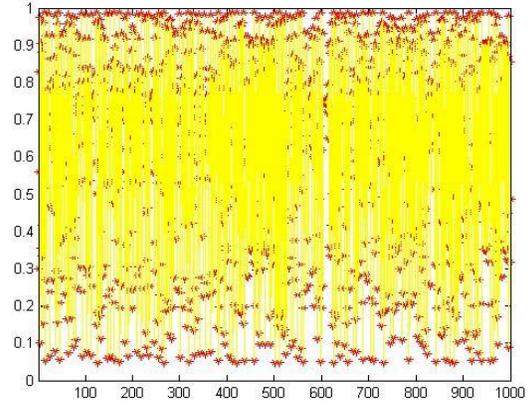
Bhupali_2(corresponding to $\lambda = 3.95$, initial point = 0.3):

GCPBSGBGGRPB SRDBRCD BPBSRCCDBRCD BPBSGCD BRPB SRDBGBRCD BGRCD BRPB SRCDBG
BRPBSSGPBSSGBRCCCCDBGCPBSRCCDBGCPBSRCD BGCPSRCCCCDCDBRPB SRDBRPBSGBRDBR
DBGCPBSRDCPB SRCDBPBSRDBGBRCD CPBSRDBRCCDCPB SGBGCPBSRCCDBGRDBRCCDCPB
RCD BGCPSRCCDCDBSPBSGSPBSRCCCCDCPB SGBGCD BRPB SRDCPB SGCPSRDBRDBRDBGSP
BSGBRDBRCD BGRCCDCPB SRDCDBRDBRCD BGRPB SRDBRDBGSPBSRCCDCDBRPB SRCDBGRD
CPBSRCD CDBRDCPB SGCDBRCCDBGCD BRPB SRDBRCD BGBSPBSGGBGGBPBSRCCDCPB SRDBGCP
SRDBR
PBSSGBGCPBSRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBR
SRDBRCCDBGBRCCCCDBGCD BRDBGCPBSGBRDCPB SRDBRCCDBRDBRDBRDBRDBRDBRDBRDBR
PBSSGBGPBSRCCDCPB SGBGRPB SRDBRCCDCPB SGCDBRPB SRCDBRCD BGPBSRDBGBRCD BRCD
CPBSRDBPBSRDBEGBRCCDCDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBRDBR
GBRCD BRDBGBPB SRDBGBRDBEGBRDCPB SGBRPBSGBCPB SGBGSPBSRCCDBRCCCCDBG
PBSRDBRCCDBPSPBSGSPBSRDBGBGCPBSRDBRDCPB SRDBRDBRDBRDBRDBRDBRDBRDBRDBR
PBSSGCPBSRDBGCPBSGSPBSRCD BPBSRDBGBGGBGGBGGBPBSRDBGBPB SRDBGBRDBRDBRDBR
DBRCD BGBGGBGGBPBSGCP

The graphical representation of the logistic maps corresponding to the given strings is shown:



Bhupali_1



Bhupali_2

Now let us try to find out close relative string of the above two set of strings.

1.2 Obtaining a Close Relative String:

The next task is to generate or evolve a new sequence from the given two sequences of the same class (same raga in present case). As said earlier the closeness is ensured by the parameter ‘Correlation Coefficient’.

Definition: Correlation Coefficient of two sequences $x_1(n)$ and $x_2(n)$ is defined as

$$c = \frac{\sum_{n=1}^{1000} x_1(n)x_2(n)}{\sqrt{E_x E_y}}$$

Where E_x and E_y respectively denotes energy of $x_1(n)$ and $x_2(n)$ respectively, i.e.

$$E_x = \sum_{n=1}^{1000} x_1^2(n), \text{ and } E_y = \sum_{n=1}^{1000} x_2^2(n)$$

Initially, the music strings are encoded to form discrete sequences where different frequencies present in the strings denote different amplitude levels of the sequence. Without loss of generality, we have chosen two discrete sequences having same number of amplitude levels (corresponding to music string of same raga) to obtain a close relative from the class.

1.2.1 The Basic Algorithm:

To get an evolved sequence from given two sequences of a given class, firstly a search space is randomly generated using typically 150000-200000 sequences of same class and same length (1000 in this case). The algorithm aids to find out the evolved or next generation sequence that maximally resembles the given 2 sequences, which can be

assured from the fact that the sum of correlation coefficients of the evolved sequence with the given two sequences is maximum. The search is random, unbiased and non adaptive as the search space is randomly generated. According to the encoded logic, the derived sequence is finally decoded to a music string (with different frequencies present) of the same raga.

2. Results:

Bhairabi:

In case of Bhairabi raga, the close relative string corresponding to *Bhairabi_1* and *Bhairabi_2* is:

gBPBMPMgPPBMScdnnMnrdrddSMgnSCCGCnrCBBRSMdrMMBCCBMnPCmdndSMSnMBCBCPBBM
nCSMnBgnnCnBgSPdBPrdCnMSgCSrdMgPSBSBrgMSCCnngndnCngMCSrMgdSMCPrnPCrBSr
nnrSPMgMSnMBSPPdrgBrCrBnSrrBMMdBdrnrgdgddrPPdrBCPMBMSSnnPBdMMrrMMSBgn
rCBgdBMSrrdPBMMgrPgrSgngCrBrMCCrndSrSdMgPdggngCrBrrgSCSrCSCSCgMPCPSgr
PdPPnBgSgCMgdrCPMSdgPBdgSSnCPggMggdngCBrsddnBPMSBnnrBCPMCddngnMgPrPCn
MdgMgnBPPgPCMPCrPMMgdnrgCdrSBcgnrrgCnSgrMSSnPMsMnPMCddgCPndBPPrMBBnrC
CSgdSBBrnMSBMPBSdPdCbCpGbnPCddPBdnPndSggdrCPddBdrnrrSPPSBCnrMgPngBCr
SnSnnPrMdrdrSnPSndnMMCdMSBMBgPSrBPBgCddrnngMnrBgrPdCdrnMndrBBMBnndCB
MSCBCMBRMMrdddMrSBnnnCPgdBMBBrrMrMBdgdnsPCndMgCdnnsPBBCnMrgBPCgMPPrPCM
rnrCgCMBPMBgMPCPdgnrMSPdCbDBMPBdrBgnMnrBrgPSMSBrndgPddMPrPMCrnBnBndCM
CgBBSggBdgdgPrgSBgggBCMSSCPnPSpSPCBrrBdCrMCMPrSBrrndnPdrPBCrgrBPMrCCS
SBnBnnCCnrCBngSdBBSnrMgrBSrgCCPdngPMgPSSBnCGCBPCCddMnndndndBPMSdBrM
CPrBrPddMCPnMddngBrCgCrnBPPnBnCrMPnPdnSMCSgMgMdndddgPPgPBMgSgnMSPBBCd
MSrrBBngBMgdSBMndPPMdrCBMBPnMdSPBnBCBrBnCSBPnngBCPSPBgPMMgMgPgMCMCBdS
BSPrdgCnCrdPdSgBMrnP

Bhupali:

The same algorithm is repeated to get the evolved sequence corresponding to *Bhupali_1* and *Bhupali_2*:

DSSSCCPGBCDSGSGRDSRGRBPSDRPPSGSCRCPDBSPBPCDCPPGSCDBSGDCSGBBCBRCSBPP
DPCCGCPBDPRBBFCCRBBCBGGSSBSGCSRRSPRBDGGRCBRDCGPGGCRPPGGPDGCCSCDD
GCGSCDGGCCSRBGGPBSPCBGSPDSRCSDDGSSSPCDPSGPGGRGCRSCGGRBDPCDRPPRDCSCDSB
PCGBDRGCSDBSGGPPGPGDGS CGCDS PGDSCDGDGBPSSRSRDPBPBGBS CDDBBGDBDRSCRBPBRP
DBSBDDCGBBBSBDCGGCPGDGPPPCRSBBCPRPCCGDRCCBBGDGSSSDGSCGSPGDRRSBGPDCDD
PBBCBCBBDDCDRDCRCBPGBPRRBDGSGCGBBRSCGCBRCGSRGCCDSGBDGBBGS SDRPDG
BBGCBCGPRPRDGDSPCBGDGRCDGPBRCGDDCGDRSGDRDDPDDGBRDBSCBPCGPDCGGRBDCR
SPCPGDPBSSCRBSBDDCCPPRPDRSRPRGRGGDRSDGSCDDBBPBCCSCGSGGCRGRBPCSBDB
DRDRCCDCCGRDSCBSCSGCCBCGGRRBSSCGBRRCRGC BPGCBBCDGDDBPCSPBGRDRPGSGCC
RBSSPPDDDCBSDGCDGPPSDRSCPDBSDCDBPSDGPDDBPBRPCDGPBCPCSPCDDPBPDDGCSB
RCCRDRGGCRGPBSRSDGCCSBGRPPDSCDDCDBSDDCRBPCGDCCPBBDGPDRRPPDPBPSBR
BPGCGPPDDDPGCRGDBSDCRGSPBGBDBSRRCPCCRPSBDSBSGCBGDDRCRDGDGDDPSPGCCRGDR
GBSSRCSBRDBPGDBGGCBGRGPCDSPGPPBRPRBRDPRCRRPDRDSRGPBDBPCCGCSPPBRPCPBS
BDCSBGCSGGDCDDCBDRDRSDDSRBDGSSBRCGDDBGSBCBPPPPBSCBCGGRDDSRBCGDBSGGC
SPSCPPDCSSRPRCDGDSBC

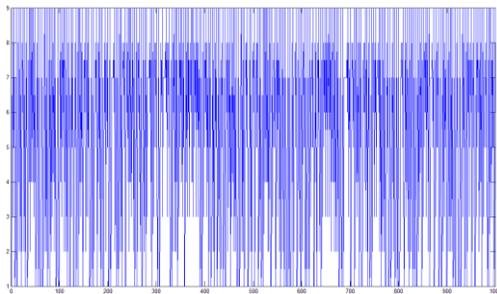
3.1 Graphical representation and Fractal Dimension:

The graphical representation of the given two strings and the generated string are shown in the following:

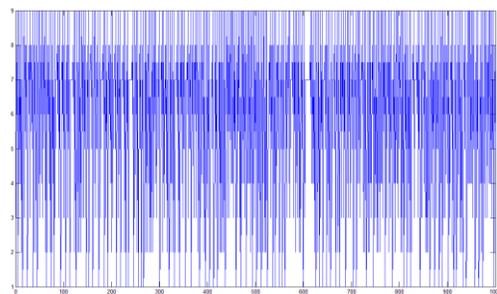
Bhairabi raga:

The discrete sequences obtained from the music strings of Bhairabi raga consist of 9 different amplitude levels corresponding to 9 different frequencies present in the string. The strings along with the derived string are plotted below:

The two strings graph:

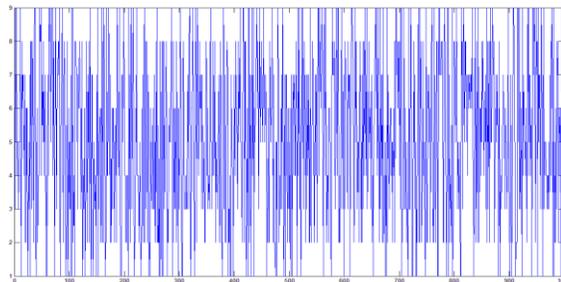


Bhairabi_1:



Bhairabi_2:

The generated string graph:

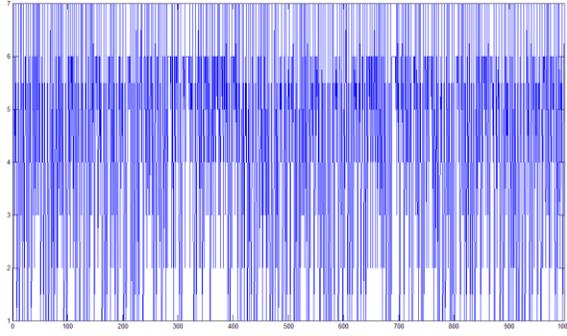


Bhairabi_1+Bhairabi_2:

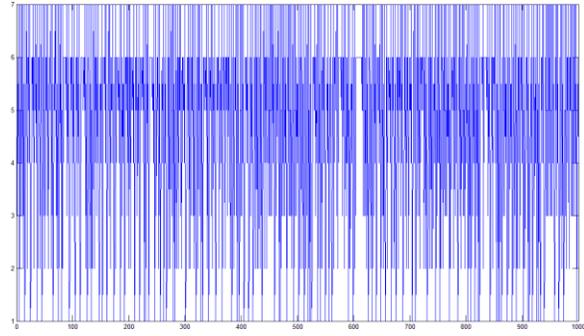
Bhupali raga:

As the strings corresponding to Bhupali raga consist of 7 different frequencies, the discrete sequences have 7 different amplitude levels. The strings along with the derived string are plotted below:

The string graphs are as follows:

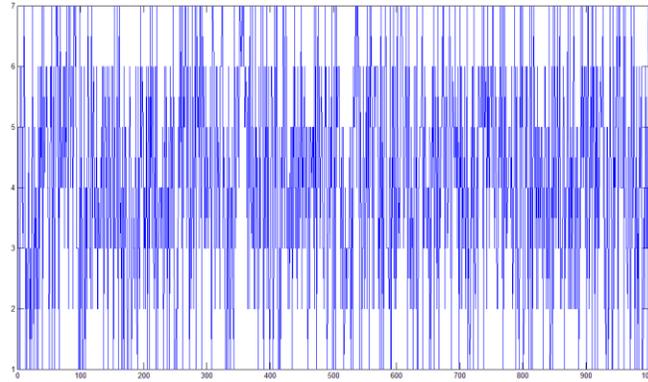


Bhupali_1:



Bhupali_2:

The generated string:



Bhupali_1+Bhupali_2:

The fractal dimensions of the graphs generated by logistic map (as shown above) are calculated. The fractal dimensions are 1.73752, 1.72622, and 1.79141 for the Bhupali_1, Bhupali_2, and Bhupali_1+Bhupali_2 respectively. The same for the strings corresponding to Bhairabi_1, Bhairabi_2, Bhairabi_1+Bhairabi_2 are 1.73161, 1.71914, and 1.79513 respectively. It is evident from the fractal dimension that the close relative string complexity is very close one of the twos.

4. Conclusion

In this paper, music strings are generated using logistic map utilizing its chaotic property and their analysis is presented. Statistical behavior of the generated strings is the object of further research. The search carried on in this paper is non-adaptive and in our future accomplishments, adaptive measures may be taken to ensure more closeness.

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