Real elimination & application to the wavelet design.

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Wavelet bases and related generating systems of $L^2(\mathbb{R}^n)$ recently became of high interest in signal/image processing, signal transmission technology and approximation theory. The main tool for constucting such objects is the multiresolution analysis (MRA) and its scaling function. This scaling function satisfies a refinement equation, which connects functional analysis and algebra.

The conditions originating in both fields are conflicting, so the solution of the resulting system of algebraic equations is crucial. Cases with low complexity were solved manually, other cases lead to solution varieties of positive dimension, which are difficult to solve even when using conventional computer algebra software (CAS).

We present a short approach to the analytical theory [Heil92, CaMo95] that allows to deduce semi–algebraic conditions [BeWa99] on the existence of the desired scaling functions. Then we explain how the theory of polar varieties [BGHM01, BGHP05] and the method of stepwise elimination as implemented in the KRONECKER–package [Lec01] was applied to the problem of constructing symmetric orthogonal and continuous scaling functions. We obtained competitive results and computational complexity.

Bibliography

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