Elimination techniques for the computation of the ideal of a smooth algebraic variety

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The estimation of upper bounds for the number of equations defining an algebraic variety has been a main object of study in algebraic geometry since the end of the nineteenth century. A result due to Kronecker (1882) states that any algebraic variety $V \subseteq \mathbb{A}^n$ can be defined by $n+1$ polynomial equations. Later, a version of this result with degree upper bounds was obtained by Heintz (1983), who proved that the defining polynomials can be chosen with degrees bounded by the degree of the variety V. Kronecker's bound was subsequently improved to n (Storch, 1972; Eisenbud-Evans, 1973), but no degree upper bound has been given in this case.

The algebraic counterpart of the previous problem concerns the study of quantitative considerations on generator sets of the ideal $I(V)$ of all polynomials vanishing on V. Even though no general upper bound depending only on n can be expected for the number of generators of $I(V)$ without additional assumptions on V, the upper bound n was proved to hold for locally complete intersection polynomial ideals (Kummar, 1978; Sathaye, 1978). However, it is not clear whether effective degree bounds (in terms of elementary geometric invariants of the variety V) for the n polynomial generators could be obtained.

We will discuss a result (joint work with C. Blanco and P. Solernó) showing that the number and degrees of polynomial generators of $I(V)$ can be controlled *simultaneously* in the case $V \subseteq \mathbb{A}^n$ is equidimensional and smooth: more precisely, by reconstructing the ideal from linear projections of V, a family of $(n - \dim(V))(1 + \dim(V))$ generators for $I(V)$ with degrees bounded by $\deg(V)$ can be obtained. In addition, we will describe a probabilistic algorithm with single exponential complexity bound for the computation of generators of $I(V)$ from a set-theoretic description of an arbitrary smooth equidimensional variety V, which we derive from the arguments used to prove the existence of the required generator set combined with the fast elimination procedures developed by the TERA group.