Computation of the singular locus of a bicubic Bezier surface.

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A common representation of surfaces in Solid Modeling and Computer Aided Geometric Design (CAGD) uses parametric bicubic patches, i.e. images of maps

$$\Phi : [0,1] \times [0,1] \to \mathbb{R}^3$$
$$(t,u) \mapsto \Phi(u,v) = \left(\frac{\Phi_1(u,v)}{\Phi_0(u,v)}, \frac{\Phi_2(u,v)}{\Phi_0(u,v)}, \frac{\Phi_3(u,v)}{\Phi_0(u,v)} \right)$$

where $\Phi_0, \Phi_1, \Phi_2, \Phi_3$ are polynomials with real coefficients and bidegree $(3,3)$. They are called Bézier when $\Phi_0(u,v) = 1$. These patches are encountered in many applications [1]. Spline surfaces are made by gluing together such patches.

The difficult step in the computation of the singular locus is to get a point on each loop of the selfintersection curve. This difficulty became a major problem in CAGD

There are many articles presenting methods and algorithms to intersect two patches (see e.g. [5], [3], [2], [4]), but very few papers address the computation of selfintersections.

We consider two distinct techniques for solving this problem. after settting a system of equations via a suited change of variables.

Either, via a bivariate resultant adapted to the corresponding elimination procedure, get a bivariate equation of bidegree $(44,44)$ describing the selfintersection locus and study it further from a computational point of view. Or via a polynomial solver for systems with floating point coefficients with a given accuracy. Examples and timings are provided.

References

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