Historical Starting Points

1) Simeón Denis Poisson, 1835 :

Recherches sur la probabilité des juguements en matière criminelle et en matière civile.

Basic result: if the mean time of arrival to a Court House is λ, then the probability of the event

{k files arrives to the Court House in a time interval of t units}

is given by

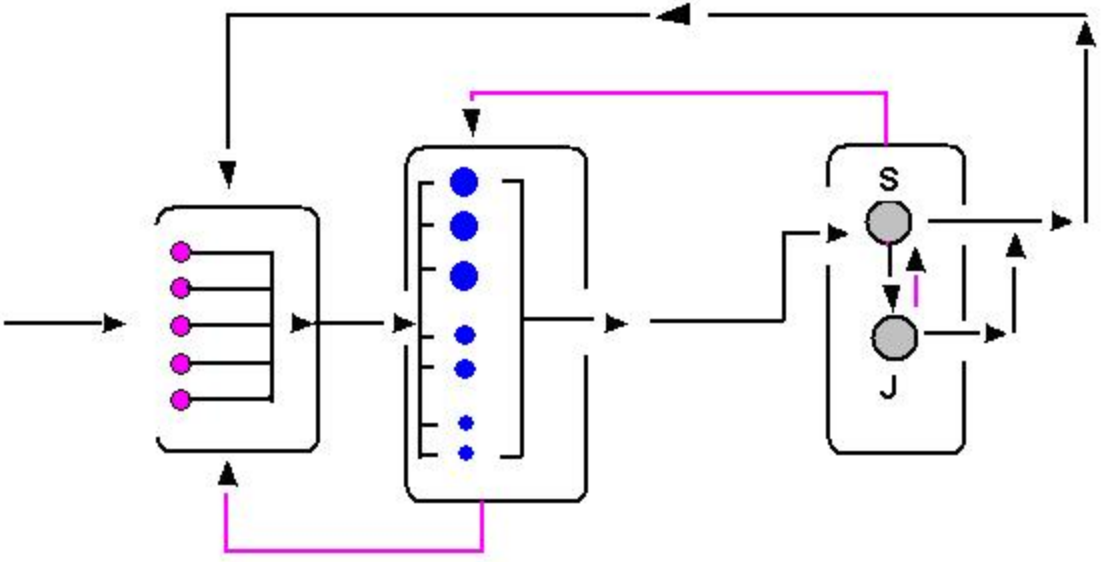
$$P(k,\lambda,t)=\frac{(\lambda t)^k}{k!}e^{-\lambda t}$$

2) Agner Erlang, 1919 :

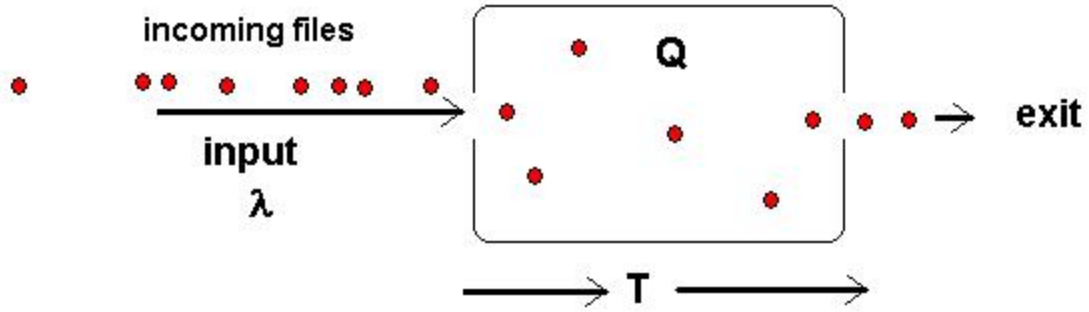
Solutions of Some Probabbility Problems of Significance for Automatic Telephone Exchanges.

In this paper, Erlang introduces the probability distribution which nowdays bears his name.

A model for a Civil Court suitable for Queueing Theoretical Techniques



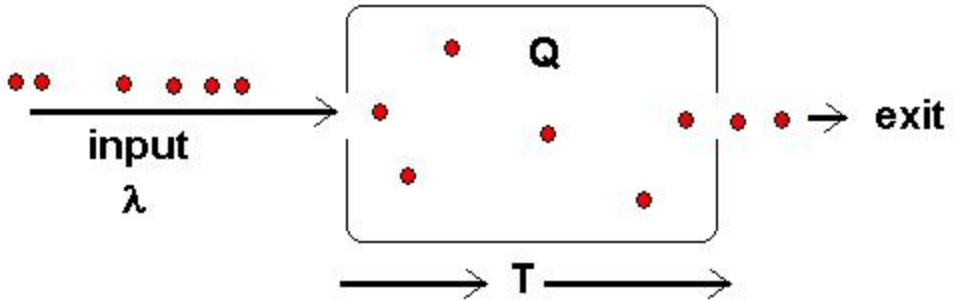
The Civil Cort House as Black Box



Fundamental Random Variables

λ	input rate
Q	number of existing files
T	legal procesal time

The Black Box is really not so black



The mean values $\overline{\lambda}$, $\overline{Q}$, $\overline{T}$, are not independent. They are linked by Little's Theorem: (1961):

$$\overline{Q} = \overline{\lambda}\ \overline{T}$$

The Chapman Kolmogorov Equations, II

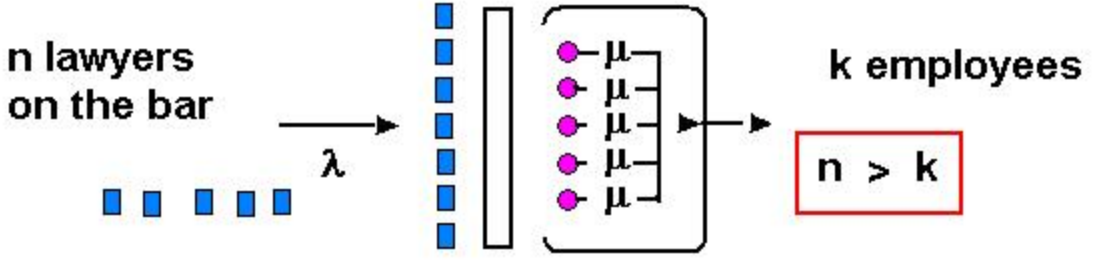
If $\sum_{j=1}^{m} p_{i,j} = 1$ *, then the matrix* $\vec{\Pi}(t)$ *is called a Markov matrix, or a stochastic matrix, or simply a probability matrix.*

Then, it can be shown, under reasonable hypotesis that

$$\frac{d\pi_i(t)}{dt} = \sum_j \pi_j(t)\,\mu_j\,p_{ji} - \pi_i(t)\,\mu_i$$

(Ch-K equations) have time dependent and steady state solutions. The latter are used in our queueing models.

A close look to the Admittance Room



The written files enter the Admittance Room with rate λ according to a Poisson process. There are n emplyees with common service rate μ. Both hypotesis are plausible and experimentally proven to be correct.

1) Are the n emplyees enough to handle ina reasonable time the incoming flow of lawyers?
2) Is the length of the bar enough for the same task?

First model available: the (M, M, n) queue
(Usual bank service queue)

The trick we found first workable was to consider the n places avalieble in the bar as intrinsec servers of the system. Whith this idea the literature provides close expresions related tho the second question in the preceeding plate.

The steady state solution of the equations that govern the system requires that

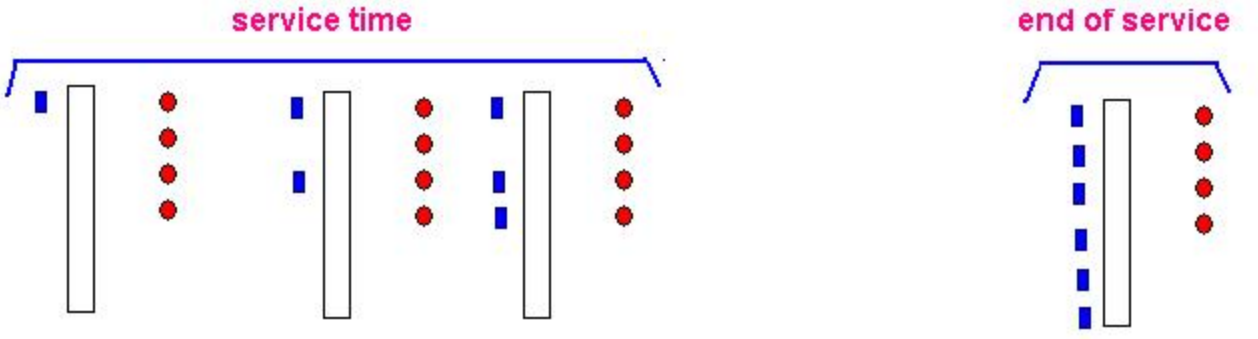
$$n > \left[\frac{\lambda}{\mu}\right] + 1$$

This inequality allows to obtain the minimum leingth for the bar.

A more realistic approach to the Admittance Room

The model that fits almost a prefect description of the Admittance Room in a Civil Court House is the so called (M, M, k, n) model. This means that we have now k servers (the existing employees) that offer service untill the queue length reaches the value n (the linear capacity of the bar). Then the system stops offering service until a lawyer in the bar leaves it. Then the system starts again.

This model is a refinement of the latter, but requires more measurements. At present time, jointly with Roberto Depaoli, we are developing the application of the model.



A possible Miró

A civil demand is established about the authenticity of the painting shown at the left. The Judge needs help, because he is not an art expert. So a couple of art specialists are officially summoned to offer an opinion.

What is the role of math in this problem? Is it possible from this point of view to offer some help? Somebody tells His Honor that he knows a mathemathician who is interested in this type of questions. Then, a third expert appears to generate more confusion.

(Klir and Folger, 1988)

Some Fuzzy Logic Tools, I.

The following sets are crisp sets, not fuzzy sets:

X is the sets of all paintings.

M is the set of all paintings by Miró.

D is the set of all the paintings of Miró's disciples.

C is the set of all couterfeits of Miró's paintings.

A basic probability assignment is a function

$$m : P(X) \to [0,\ \ 1] \qquad \text{such that}$$

$$m(\varnothing) = 0 \qquad \text{and} \qquad \sum_{A \in P(X)} m\,(A) = 1$$

(Observe that m IS NOT a probability distribution)

Some Fuzzy Logic Tools, II

Then, given a basic assignement m a belief measure is uniquely determined by

$$Bel(A) = \sum_{B \subseteq A} m(B)$$

i) $m(B)$ **is interpreted as the degree of evidence supporting the claim that a specific element of X belongs to B, but not to any special subset of B.**

ii) $Bel(A)$ **represents the total evidence or belief that the element in question belongs to the set A, as well to the various special subsets of A.**

(Bel (A) is an example of a fuzzy measure)

Some Fuzzy Logic Tools III

Then , evidence obtained by two independent art experts are expressed by two probability assignements m_1 and m_2, on some power set P(X). These assignements produce a joint basic assignement $m_{1,2}$ by means of Dempster's formula:

$$m_{1,2}(A) = \frac{\sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1 - K}$$

where $A \neq \varnothing$ **and** $K = \sum_{B \cap C = \varnothing} m_1(B) \cdot m_2(C)$