Fitting data with shift invariant spaces.

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Given data $f_1, \ldots, f_m$ in $L^2(\mathbb{R})$ with m usually large, and a fixed integer $n \leq m$, we study the problem of whether there exists a shift invariant space with exactly n generators, which is an optimal model for these data. That is, we look for a subspace that best fits the data in the least square sense.

More precisely, denote by $V_n = V_n(\Phi)$ a shift invariant space of $L^2(\mathbb{R})$ with a set of n generators $\Phi = (\phi_1, \ldots, \phi_n)$ whose integer translates form a Riesz sequence. Let $E_n = E_n(f_1, \ldots, f_m, V_n) = \sum_{i=1}^{m} \|f_i - P_{V_n}(f_i)\|^2$. The problem under study is: if there exists V_n that minimizes E_n for the data $f_1, \ldots, f_m$ where the minimum is taken over all possible shift invariant spaces V_n' of $L^2(\mathbb{R})$. We prove that such subspace always exist, and we provide a method to construct a set of generators from the data. In this talk we will present this result and we will also describe some extensions and related problems.