Cornucopia of Randomness

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There is general consensus in the literature to adopt Martin Löf 's definition of randomness for real numbers (a real number is random if it belongs to no constructible G_δ set of measure 0). Although the set of random reals has Lebesgue measure 1, giving examples has been a rather difficult affair. The first example is Chaitin's number Ω, the probability that a universal computer halts (1975).

Inspired by Chaitin's work, I will present a theorem that acts as a source of random numbers, which occur as probabilities of certain computer behavior. Once we formalize possibly infinite computations as effective continuous maps in appropriate topological spaces, the probability that a computer gives an output in a given set A becomes the Lebesgue measure of the inverse image of A by that map. Our theorem gives sufficient conditions on an output set A for the probability that a universal computer produces an output in A to be random. Moreover, the harder the set A, the more random the probability.

Joint work with Serge Grigorieff.