Complexity of Integer Quasiconvex Polynomial Optimization.

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The following case of integer polynomial optimization is considered:
Minimize a polynomial $\hat{F}$ on the set of integer points described by an inequality system $F_1 \leq 0, \ldots, F_s \leq 0$, where $\hat{F}, F_1, \ldots, F_s$ are quasiconvex polynomials in n variables with integer coefficients.
An algorithm solving this problem is designed that belongs to the time-complexity class $O(s) \cdot l^{O(1)} \cdot d^{O(n)} \cdot 2^{O(n^3)}$, where $d \geq 2$ is an upper bound for the total degree of the polynomials involved and l denotes the maximum binary length of all coefficients. The algorithm is polynomial for a fixed number n of variables and represents a direct generalization of H.W.Lenstra's algorithm [3] in integer linear optimization. Our complexity–result improves as well the one due to Bank et al. [1] as the one due to Khachiyan and Porkolab [2] for integer polynomial optimization in the considered case.

References

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[2] L. Khachiyan, L. Porkolab. *Integer optimization on convex semialgebraic sets*, in *Discrete & Computational Geometry*, 23:207-224. Springer-Verlag, 2000.

[3] H. W. Lenstra. *Integer programming with a fixed number of variables*, in *Mathematics of Operations Research*, 8:538-548, 1983.