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**Comptage des composantes connexes d'un ensemble semi-algébrique en temps simplement exponentiel. (French. English summary) [Counting connected components of a semi-algebraic set in single exponential time]**

*C. R. Acad. Sci. Paris Sér. I Math.* **311** (1990), *no. 13*, 879–882.
14P10 (14Q99 68Q25)



This paper is an announcement of the following result: Let $S \subset \mathbf{R}^n$ be a semialgebraic set defined by $s$ polynomials of degree $\leq d$ and coefficients of binary length $\leq t$. Then it is possible to decide in $[s \cdot \sup(d,t)]^{n^{O(1)}}$ time if two points $x$ and $x'$ belong to the same connected component of $S$. It is also possible in the same time to count the number of connected components of $S$. Complete proofs will appear in papers by Grigor′ev and Vorob′ev ["Counting connected components of a semi-algebraic set in subexponential time", in *Computational complexity*, to appear] and by Heintz, Roy and Solernó [in *Applied algebra, algebraic algorithms and error correcting codes* (Tokyo, 1990), 180–196, Lecture Notes in Comput. Sci., 508, Springer, Berlin, 1991; Part II, to appear].

The paper contains indications on the general lines of the proof and on the methods used. It also contains some comments on related results, in particular on those of J. Canny [*The complexity of robot motion planning*, MIT Press, Cambridge, MA, 1988; MR0952555 (89m:68142)].

**Reviewed** by *Robert Silhol*

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**Comptage des composantes connexes d'un ensemble semi-algébrique en temps simplement exponentiel. (Counting connected components of a semi- algebraic set in single exponential time).** (French)

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The paper gives a description of some results on the single exponential complexity of the counting of the connected components of a semialgebraic set $S \subset \mathbb{R}^n$, and explains the main ideas of the proofs. In fact, similar results had been independently obtained by the different authors and essentially following the same lines. Thus somehow they decided to write a common presentation emphasizing where their approaches avoid the double exponential complexity known till now. — The main idea is to project directly to $\mathbb{R}$ (instead of an iterative projection lowering the dimension in each step) and to construct from this a semialgebraic curve on $S$ with the property that it intersects each connected component in a connected arc. The subtle point is the analysis of the complexity of the process, for which the reader is referred to the original papers.

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*Classification* :

∗14P10 Semialgebraic sets and related spaces

14Q99 Computational aspects of algebraic geometry

68Q25 Analysis of algorithms and problem complexity

Cited in ...