A Fleming-Viot process driven by sub-critical branching: a selection principle.

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Joint work with
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Absorbing processes

$Z = (Z_t, t \geq 0)$, a pure jump Markov process in $\Lambda \cup \{0\}$

$Q = (q(x, y), x, y \in \Lambda \cup \{0\}$ the rates.)

Λ is an irreducible class.

μ is the intial ditribution.

0 is absorbing (i.e. $q(0, y) = 0$ for all $y \in \Lambda$)

Absorption is certain: $\mathbb{P}_\mu(Z_t = 0, \text{for some } t > 0) = 1$.

Unique invariant distribution: δ_0. In this case we study the conditioned evlution

$$\varphi_t^\mu(x) = \frac{\mathbb{P}_\mu(Z_t = x)}{\mathbb{P}_\mu(Z_t \neq 0)}$$

Quasi-stationary distributions

φ^μ is the unique solution to the Kolmogorov forward equations

$$\frac{d}{dt}\varphi^\mu_t(x) = \sum_{y\in\Lambda} q(y,x)\,\varphi^\mu_t(y) + \sum_{y\in\Lambda} q(y,0)\,\varphi^\mu_t(y)\,\varphi^\mu_t(x).$$

The *Yaglom limit* for the measure μ is defined by

$$\lim_{t\to\infty} \varphi^\mu_t(y), \quad y \in \Lambda,$$

if the limit exists and is a probability on Λ.
A *quasi-stationary distribution* (QSD) for Q is a probability measure ν on Λ that is invariant under $\{\varphi_t, t \geq 0\}$, that is

$$\varphi^\nu_t = \nu, \quad \text{for all} \quad t \geq 0.$$

If the Yaglom limit exists, it is known to be a QSD (and a QSD is a Yaglom limit).

Some difficulties

Non-linear semigroup.

It is non-attractive, even if Z is.

Markov process theory can not be applied.

In particular: the number of quasi-stationary distributions can be 0, 1 or ∞.

There is no obvious way to simulate neither the QSD nor the conditioned evolution for large times.

Example: linear birth and death process

$$q(x, x+1) = px, \qquad q(x, x-1) = (1-p)x$$

There is a one parameter family of QSD if $p < \frac{1}{2}$, and no one if $p = \frac{1}{2}$. (Seneta-Vere-Jones, 1966. Cavender, 1978)

Some intuition

Let T be the aborption time and assume $\Lambda = \mathbb{N}$.

Theorem (Ferrari, Kesten, Martínez and Picco 1995):
Assume $\lim_{x\to\infty} \mathbb{P}_{\delta_x}(T < t) = 0$*, then*

$$\textit{There exists a } \mathrm{QSD} \iff \mathbb{E}(e^{\theta T}) < \infty, \ \textit{for some } \theta > 0.$$

The Fleming-Viot process (FV) driven by Q

We have N particles, each particle moves independently of the others as a continuous time Markov process with rates Q, but when it attempts to jump to state 0, it comes back immediately to Λ by jumping to the position of one of the other particles chosen uniformly at random.

Denote

$\xi_t = \xi_t^{N,\xi^0} = (\xi_t(1), \ldots, \xi_t(N)) \in \Lambda^N$ the state of the process at time t.

$\eta(\xi, x)$ the number of ξ particles at site x.

$m_x(\xi) = \frac{\eta(\xi,x)}{N}$, $x \in \Lambda$ the empirical measure.

The Fleming-Viot process (FV) driven by Q

Generator

$$\mathcal{L}^N f(\xi) = \sum_{i=1}^{N} \sum_{x \in \mathbb{N} \setminus \{\xi(i)\}} \left[q(\xi(i), x) + q(\xi(i), 0) \, \frac{\sum_{j \neq i}^{N} \mathbf{1}_{\{\xi(j) = x\}}}{N - 1} \right] (f(\xi^{i,x}) - f(\xi)),$$

$$\xi^{i,x}(j) = \begin{cases} x & j = i, \\ \xi(j) & j \neq i, \end{cases}$$

FV as an approximation of the conditioned evolution and QSD

We have

$$\frac{d}{dt}\mathbb{E}\frac{\eta_t(x)}{N} = \sum_{y\in\Lambda} q(y,x)\mathbb{E}\frac{\eta_t(y)}{N} + \sum_{y\in\Lambda} q(y,0)\mathbb{E}[\frac{\eta_t(y)}{N}\frac{\eta_t(x)}{N-1}].$$

Recall

$$\frac{d}{dt}\varphi_t^{\mu}(x) = \sum_{y\in\Lambda} q(y,x)\,\varphi_t^{\mu}(y) + \sum_{y\in\Lambda} q(y,0)\,\varphi_t^{\mu}(y)\,\varphi_t^{\mu}(x).$$

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In the countable state space setting
Ferrari, Marić, Asselah, Jonckheere

Problems (some)

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Remark: FV inherits the difficulties of the conditioned process.

Problem 1: Particles correlations

Evolution of the empirical profile

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Conditioned evolution

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So $e_t(x) = \mathbb{E}\frac{\eta_t(x)}{N} - \varphi_t^\mu(x)$ verifies

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$$R(\xi;x,t) = \sum_{y\in\Lambda} q(y,0)\left[\frac{N}{N-1}\mathbb{E}\left[m_y(\xi_t)m_x(\xi_t)\right] - \mathbb{E}m_y(\xi_t)\mathbb{E}m_x(\xi_t)\right].$$

Proposition (Ferrari-Marić, 2006)
For each $t > 0$, and any $x, y \in \Lambda$

$$\sup_{\xi \in \Lambda^N} \left| \mathbb{E}^{\xi}[m_x(\xi_t) m_y(\xi_t)] - \mathbb{E}^{\xi}[m_x(\xi_t)] \, \mathbb{E}^{\xi}[m_y(\xi_t)] \right| \leq \frac{e^{Ct}}{N}.$$

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Proof. Follows from

$$\left| \mathbb{P}(\xi_t(i) = x,\, \xi_t(j) = y) - \mathbb{P}(\xi_t(i) = x)\, \mathbb{P}(\xi_t(j) = y) \right| \leq \frac{e^{Ct}}{N}.$$

which can be proved by coupling.

Coming back to the conditioned evolution...

$$e_t(x) = \mathbb{E}\frac{\eta_t(x)}{N} - \varphi_t^\mu(x)$$

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Coming back to the conditioned evolution...

Under reasonable assumptions on Q, Gronwall's inequality gives us

$$\mathbb{E}(\|m(\xi_t) - \varphi_t^{m(\xi_0)}\|_2) \leq \frac{e^{Ct}}{N}$$

i.e. For compact time intervals, the empirical measure of FV converges to the conditioned evolution

Subcritical branching

Let $\{p(n),\ n \in \mathbb{N}\}$ be the offspring distribution. We consider $\Lambda = \mathbb{N}$ and $\{q(x,y); x, y \in \mathbb{N}\}$ of the form

$$q(x, x+i-1) = xp(i), \quad i \neq 1, \quad q(x,x) = -x,$$

and $q(x,y) = 0$ otherwise.
We assume

$$-v := \sum_{i=-1}^{\infty} ip(i+1) < 0 \qquad \text{(and exponential moments)}$$

There is a one parameter family of QSD! (Seneta-Vere-Jones 1966, Cavender 1978, Van Doorn 1991)

Subcritical branching

The minimial QSD ν_{min} has generating function given by

$$G(\nu_{\mathsf{min}}; z) = 1 - \exp\left(-v \int_0^z \frac{du}{\sum_{i\geq 0} p(i)u^i - z}\right),$$

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Also it has the minimum expected absorption time.

Subcritical branching

Theorem. (Asselah, Ferrari, G., Jonckheere)
For each $N \geq 1$, the Fleming-Viot process driven by subcritical branching is ergodic with invariant measure λ^N and for each $x \in \mathbb{N}$ we have

$$\lim_{N\to\infty} \int |m_x(\xi) - \nu_{\min}(x)| \, d\lambda^N(\xi) = 0.$$

Strategy of proof

$$\int |m_x(\xi) - \nu_{\min}(x)| \, d\lambda^N(\xi) = \int |m_x(\xi_t^\xi) - \nu_{\min}(x)| \, d\lambda^N(\xi)$$

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$$\leq \sup_{K(\alpha)} |m_x(\xi_t^\xi) - \varphi_t^{m(\xi)}(x)| + \sup_{K(\alpha)} |\varphi_t^{m(\xi)}(x) - \nu_{\min}(x)| + 2\lambda^N(K(\alpha)^c)$$

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$$(iv) \lambda^N(K(\alpha)^c) \le \frac{\kappa}{\alpha}$$

Proposition. *The Fleming-Viot process can be embedded in a Multitype Branching Markov Chain (MBMC) driven by Z but avoiding the jumps to 0 ($\tilde{Z}$).*

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Large deviations, another coupling and drift inequalities

Proposition. *The Fleming-Viot process can be embedded in a Multitype Branching Markov Chain (MBMC) driven by Z but avoiding the jumps to 0 ($\tilde{Z}$).*

Corollary. The bounds obtained for the (reflected) driving process $\tilde{Z}$ also hold for the Fleming-Viot process but with a factor e^{Ct}.

Graphical construction II

The Multitype Branching Markov Chain

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We have a marked PP ω_i^V with rate $C := \sup q(x, 0)$ for each particle. The marks are uniform in $[0, 1] \times \{1, \ldots, N\} \setminus \{i\}$

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Individuals evolve spatially according to $\tilde{Z}$.

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If ω_i^V rings and the mark $U \leq q(\xi(i), 0)/C$ then particle i (is absorbed and) jumps over the type i individual branched at that time from particle j.

Large deviations for $\tilde{Z}$ and

Proposition. Let $\delta \geq 1$ and a time T such that $Tp(0) \leq \delta/4$. Then

$$\mathbb{P}\left(\sup_{s<T} |\tilde{Z}(x;s) - e^{-vs}x| \geq \delta\right) \leq \mathrm{e}^{-C_T \frac{\delta^2}{\max\{x,\delta\}}}$$

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Corollary.

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Corollary.

(i) Fleming-Viot driven by subcritical branching is ergodic for each N.

(ii) We have drift inequalities for $\max_i \xi_t^2(i)$

Open problems

Same result for general Q.
In particular, is open when Q is a birth and death process with constant rates.

THANKS.