Practica 8. Procesos Markovianos de salto (FG).
Procesos Estocásticos 2015.

1. Exercises

Exercise 1 Let X_t be a continuous time process in $G = \{0, 1\}$ with rates $q(0, 1) = 1$, $q(1, 0) = 2$. Compute the Kolmogorov equations and find $p_t(1, 0)$.

Exercise 2 Coupling of pure birth processes. Prove that it is possible to couple two pure birth processes (X^1_t) and (X^2_t) with rates $\lambda^-_1 \geq \mu_2$, respectively, in such a way that if $X^1_0 \geq X^2_0$, then $X^1_t \geq X^2_t$.

Exercise 3 Find a positive recurrent process with a null recurrent skeleton.

Exercise 4 Consider a system consisting of one queue and one server. The clients arrive at rate λ^+ in groups of two. The system serves one client at a time at rate λ^-. Assume the system has a maximal capacity of 4 clients. That is, if at some moment there are 3 clients and a group of two arrives, then only one of those stays in the system and the other is lost. The space state is $G = \{0, 1, 2, 3, 4\}$ and the transition rate matrix of such a system is given by

$$Q = \begin{pmatrix} -\lambda^+ & 0 & \lambda^+ & 0 & 0 \\ \lambda^- & -\lambda^- + \lambda^+ & 0 & \lambda^+ & 0 \\ 0 & \lambda^- & -\lambda^- + \lambda^+ & 0 & \lambda^+ \\ 0 & 0 & \lambda^- & -\lambda^- + \lambda^+ & \lambda^+ \\ 0 & 0 & 0 & \lambda^- & -\lambda^- \end{pmatrix} \tag{5}$$

(a) Establish the balance equations and find the invariant measure.
(b) Compute the probability that a group of two people arrive to the system and none of them can stay in it.
(c) Compute the mean number of clients in the system when the system is in equilibrium.
(d) Compute the mean time a client stays in the system.

Exercise 6 Consider a queue system M/M/∞, that is, arrivals occur according to a Poisson process of rate λ^+ and service times are exponentially distributed with rate λ^- but now the system has infinitely many servers (that is all clients start service upon arrival). Solve items of the previous exercise in this case.

Exercise 7 Consider a population with m individuals. At time zero there are k "infected" individuals and $m - k$ non infected. An infected individual heals after an exponentially distributed time with parameter λ^-. If there are k infected individuals, the rate for each one of the remained $m - k$ non infected individuals to get infected is $\lambda^+(k + 1)$.
(a) Establish the balance equations.
(b) For $m = 4$, $\lambda^+ = 1$ and $\lambda^- = 2$ compute the invariant measure.
(c) Compute the average number of infected individuals under the invariant measure.
(d) Compute the probability of the event "all individuals are infected" under the invariant measure.