Gibbs point processes

1. Consider the free birth and death process $(\tilde{\eta}_t)_{t\in\mathbb{R}}$ constructed as a function of the Poisson process $\mathbf{C}$. Show that the marginal distribution of η_t is a Poisson process in $\mathbf{G}$ with intensity $w(d\gamma)$.

2. Consider the birth and death process $(\tilde{\eta}_t^\eta)_{t\in\mathbb{R}^+}$ with $\eta_0 = \eta$, a fixed locally finite configuration, constructed with cylinders $\mathbf{C}\cup\mathbf{C}_0^\eta$, see notation in the notes. Show that the distribution of $\tilde{\eta}_t^\eta$ converges to the Poisson process of the previous exercise.

3. Describe the specifications of the seed+grain process.

4. Consider the birth and death process in $\mathbb{R}^d$ with rates $M(\gamma|\eta)$ of birth and rate 1 of death. Take $M(\gamma|\eta) = \mathbf{1}\big\{\eta\cup\{\gamma\}\in A_\Lambda\big\}$, where

$$A_\Lambda = \{\eta : \|x-y\| > 2, \text{ for all } x, y \in \eta\cap\Lambda\} \tag{1}$$

   Show that $\mu_\Lambda := \tilde{\mu}(\cdot|A_\Lambda)$ is reversible for the dynamics.

5. Show that if $T \sim$Exponential(λ), then $P(T\in[t,t+h)|T>t) = \lambda h + o(h)$.

6. Let $\tilde{\eta}_t$ be the free birth and death process with parameter λ and let Λ be a bounded region. For $t\in\mathbb{R}$ define $\tau(t) = \sup\{s\le t : \tilde{\eta}_s\cap\Lambda = \emptyset\}$; this is a function of $(\tilde{\eta}_t)_{t\in\mathbb{R}}$. Show that $\tilde{\mu}(\eta : \tau(t) > -\infty) = 1$.

7. Use the graphical construction to show that if the clan of ancestors of a given cylinder is finite with probability one, then the thermodynamic limit follows. That is,

$$\lim_{\Lambda\nearrow\mathbb{R}^d} \mu_\Lambda = \mu, \tag{2}$$

   where μ is the distribution of η_t obtained from the graphical construction.