1. Prove the Disjointness Lemma. Let S_1 and S_2 Poisson processes on R and A measurable with $\mu_1(A)$ and $\mu_2(A)$ finite. Then $P(S_1 \cap S_2 \cap A = \emptyset) = 1$.

2. Construct a homogeneous Poisson process in $\mathbb{R}^d$ with intensity $\lambda \in \mathbb{R}_+$ as a union of Poisson processes on B_i, a partition of $\mathbb{R}^d$ with $|B_i| < \infty$.

3. Let μ be a mean measure on $\mathbb{R}^d$ with intensity $\lambda(x)$ and $\hat{S}$ be a homogeneous Poisson process of intensity 1 on $\mathbb{R}^d \times \mathbb{R}_+$. Define
$$S = \{x \in \mathbb{R}^d : (x, y) \in \hat{S},\ 0 \le y \le \lambda(x)\} \tag{1}$$
Prove that S is a Poisson process on $\mathbb{R}^d$ with intensity $\lambda(\cdot)$.

4. Prove the restriction theorem: Let S be a Poisson process on the space R with mean measure μ and $B \subset R$. Then $S_B := S \cap B$ is a Poisson Process with mean measure $\mu_B(A) = \mu(A \cap B)$.

5. Prove the mapping theorem: Let S be a Poisson process on the space R with mean measure μ. Let $f : R \to T$ be a measurable function such that
$$\mu^*(B) := \mu(f^{-1}(B)) \tag{2}$$
has no atoms. Then $S^* := f(S)$ is a Poisson process on T with mean measure μ^*.

6. Let S be homogeneous Poisson process with mean measure $\mu(A) = |A|$. $\lambda : \mathbb{R} \to \mathbb{R}_+$ an intensity. Take $f = \lambda^{-1}$. Show that
$$S^* = f(S) \quad \text{is a Poisson process with intensity } \lambda.$$

7. Let S be a PP with constant intensity λ on $\mathbb{R}^2$. Let
$$f(x, y) = ((x^2 + y^2)^{1/2}, \tan^{-1}(y/x)), \tag{3}$$
Prove that $S^* = \{f(s) : s \in S\}$ is a Poisson process on $\{(r, \theta) : r > 0, 0 \le \theta < 2\pi\}$ with intensity $\lambda^*(r, \theta) = \lambda r$.

8. Assume Campbell formula holds for positive f. Prove it for all f by decomposing $f = f^+ - f^-$.

9. Let S be a Poisson process in $\mathbb{R}^d$ with intensity $\lambda(\cdot)$. Let $(X_s : s \in S)$ be a sequence of iid random variables. Show that the process
$$\tilde{S} := \{s + X_s : s \in S\} \tag{4}$$
is a Poisson process with absolutely continuous measure $\tilde{\mu}$. Compute the intensity of $\tilde{\mu}$.

   Show that if $\lambda(x) \equiv \lambda$, then $\tilde{S} \overset{D}{=} S$.

10. Let $(X_i : i \in \mathbb{Z}^d)$ be a family of iid random variables with distribution $\mathcal{N}(0, \sigma^2 I)$, where I is the identity matrix. Let
$$S_\sigma := \{i + X_i : i \in \mathbb{Z}^d\} \tag{5}$$
Prove that as $\sigma \to \infty$, S_σ converges to a homogeneous Poisson process with intensity 1.