$$\sum_{k=0}^{n} \binom{2n+1}{k}$$

$$\sum_{k=0}^{m} \binom{m}{k} = \sum_{k=0}^{m} \binom{m}{k} 1^k 1^{n-k} = (1+1)^m = 2^m$$

$$m = 2n+1$$

$$\sum_{k=0}^{2n+1} \binom{2n+1}{k} = 2^{2n+1}$$

$$\sum_{k=0}^{n} \binom{2n+1}{k} + \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} = 2^{2n+1}$$

$$\binom{m}{k} = \binom{m}{m-k}$$

$$\sum_{k=n+1}^{2n+1} \binom{2n+1}{k} = \sum_{k=n+1}^{2n+1} \binom{2n+1}{2n+1-k}$$

$$j = 2n+1-k$$

si $k = n+1$ entonces $j = 2n+1-(n+1) = 2n-n = n$
si $k = 2n+1$entonces $j = 0$

$$\sum_{k=n+1}^{2n+1} \binom{2n+1}{2n+1-k} = \sum_{j=n}^{0} \binom{2n+1}{j} = \sum_{j=0}^{n} \binom{2n+1}{j}$$

$$2\sum_{k=0}^{n} \binom{2n+1}{k} = 2^{2n+1}$$

$$\sum_{k=0}^{n} \binom{2n+1}{k} = 2^{2n+1}/2 = 2^{2n}$$