Non-linear homogenization. Modeling composites

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The stresses that an ideally plastic material can withstand form a bounded closed set in the space of symmetric 3×3 real matrices with the standard distance. This set, which is a material property, is called the yield set. Unlike brittle materials, ideally plastic materials do not break. When subjected to a stress that is in the boundary of the yield set, the material experiences a permanent deformation, called plastic deformation.

Fiber reinforced composites are materials made of solid fibers embedded in a weaker solid referred to as the matrix. In this work, the fibers are assumed to be infinite in length and parallel. Thus, symmetry allows the study to be restricted to a plane perpendicular to the fibers.

The microstructure of the composite refers to the description of the regions in space occupied by the fibers and the matrix. It is described by a characteristic function χ, $\chi(\mathbf{x}) = 1$ if $\mathbf{x}$ is inside a fiber and $\chi(\mathbf{x}) = 0$ otherwise. The microstructures are assumed to be periodic, i.e. χ is $[0,1]^2$-periodic. The consideration fiber reinforced composites where both the fibers and the matrix are made of ideally plastic solids with a different yield set leads to the following homogenization problem:

Let M represent the relative strength of the fibers with respect to the matrix. It is assumed that $M \gg 1$. A two-dimensional vector field σ is said to be admissible if it is $[0,1]^2$-periodic, it satisfies the restrictions $\|\sigma(\mathbf{x})\| \leq M\chi(\mathbf{x}) + 1 - \chi(\mathbf{x})$ and the equilibrium equation $\nabla \cdot \sigma = 0$. The goal is to compute the set $\mathbf{Y}_{\text{hom}} = \{\tau : \tau = \langle\sigma\rangle$, for some admissible vector field $\sigma\}$, where $\langle\sigma\rangle$ denotes the average of σ. A new and sharp bound on the set $\mathbf{Y}_{\text{hom}}$ is obtained in this work.