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A Non-classical Heat Conduction Problem with a Source Depending of the Total Heat Flux on the Boundary

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ABSTRACT

Motivated by the modeling of the temperature regulation within a medium we consider the non-classical heat conduction equation in a semi-space n-dimensional domain $D=\mathbb{R}^{+}\times\mathbb{R}^{n-1}$ for which the internal energy supply depends on the total heat flux in the time variable on the boundary $S=\partial D=\{0\}\times\mathbb{R}^{n-1}$, with homogeneous Dirichlet boundary condition and an initial condition. The problem consists in finding the temperature $u=u(x,t)$ such that the following conditions are satisfied:

$$\begin{cases} i)\; u_t-\Delta u=-F\left(\int_0^t u_x(0,y,s)\,ds\right), & x>0,\quad y\in\mathbb{R}^{n-1},\quad t>0 \\ ii)\; u(0,y,t)=0, \quad y\in\mathbb{R}^{n-1},\quad t>0; & \\ iii)\; u(x,y,0)=h(x,y), \quad x>0,\quad y\in\mathbb{R}^{n-1} & \end{cases}$$

By using a Volterra integral equation of second kind in the time variable with a parameter $y\in\mathbb{R}^{n-1}$ the solution to this problem is obtained. The solution to that Volterra integral equation is the heat flux on S, which is an additional unknown of the considered nonlinear problem. We show that a unique local solution exists, which can be extended globally in time.

Finally, a one-dimensional case is studied and we obtain the explicit solution by using the Adomian method and we derive its properties. We must use a double induction principle in order to obtain that explicit solution which is also related to the Mittag-Leffler functions. Moreover, we obtain a relationship between this solution with a third order ordinary differential equation with a singular second member, with two initial conditions at the fixed boundary and an integral boundary condition within the domain.