Games for PDEs with eigenvalues of the Hessian

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Abstract

For a function $u : \Omega \subset \mathbb{R}^N \mapsto \mathbb{R}$, we consider the Hessian, D^2u, and its ordered eigenvalues

$$\lambda_1(D^2u) \leq ... \leq \lambda_N(D^2u).$$

Here our main concern is the Dirichlet problems for the equations:

$$P_k^+(D^2u) := \sum_{i=N-k+1}^{N} \lambda_i(D^2u) = 0, \tag{1}$$

(note that P_k^+ is just the sum of the k largest eigenvalues)

$$P_k^-(D^2u) := \sum_{i=1}^{k} \lambda_i(D^2u) = 0, \tag{2}$$

(P_k^- is the sum of the k smallest eigenvalues) and, more generally, any sum of k different eigenvalues

$$P_{i_1,...,i_k}(D^2u) := \sum_{i_1,...,i_k} \lambda_{i_j}(D^2u) = 0. \tag{3}$$

These operators appear in connection with geometry but our goal is to provide a probabilistic interpretation.

We will describe games whose values approximate viscosity solutions to these equations in the same spirit as the random walk can be used to approximate harmonic functions.

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