Logarithmic corrections in Fisher-KPP problems for the Porous Medium Equation

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We consider the large time behaviour of solutions to the Porous Medium Equation with a Fisher-KPP type reaction term

$$u_t = \Delta u^m + u - u^2 \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+, \qquad u(\cdot, 0) = u_0 \quad \text{in } \mathbb{R}^N,$$

$m > 1$, for nonnegative, nontrivial, radially symmetric, bounded and compactly supported initial data u_0. It is well known that in spatial dimension one there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave solution Φ_{c_*} with a finite front. We prove that there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty} \left\{ \sup_{x\in\mathbb{R}^N} |u(x,t) - \Phi_{c_*}(|x| - c_* t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). Moreover, the radius, $h(t)$, of the support of the solution at time t satisfies

$$\lim_{t\to\infty} \left[h(t) - c_* t + (N-1)c^* \log t \right] = r_0.$$

Thus, in contrast with the semilinear case $m = 1$, we have a logarithmic correction only for $N > 1$.

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