Multidimensional optimization of dividends: a problem with free regions

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Abstract

In [1], we considered the problem of maximizing the expectation of discounted dividends until a stopping time. This stopping time is defined as the minimum between the ruin time (that is when the reserve of the company becomes negative) and a decision time that stops the process (with an extra dividend payment at that time which depends on the reserve by a function f) . The uncontrolled reserve follows a simple stochastic jump process (compound Poisson with negative jumps). This is a mixed singular control/optimal stopping problem. In the present work, we generalize [1] to a multidimensional setting (n associated companies). As in the unidimensional case, it can be proved that the optimal value function V can be characterized as the smallest viscosity supersolution of the associated non-linear integro-differential Hamilton-Jacobi-Equation (it is also a viscosity solution of this equation).The HJB equation is a maximum between $n+2$ operators equal to zero. The $n+2$ regions of the state space $\mathbf{R}_+^n$ in which each operator applied to V is zero determine the optimal strategy: One operators is $V-f$ and so f can be regarded as an obstacle (the corresponding set is associated to the decision region), another operator is an integro-differential one (arising from the jump process) and the corresponding set gives the non-action region, the other n operators are related to the regions in which each company pays dividends. The hardest part of the problem is to find these $n+2$ regions. The main contribution of this work is to approximate these regions and V by a numerical procedure; we propose to consider a family of admissible strategies (based on the $n+2$ operators of the HJB equation) in a suitable grid that satisfy a discrete version of the HJB equation and show that the value function of these strategies converge locally uniformly to the optimal value function V as the mesh size goes to zero. We also show examples. This is a joint work with Pablo Azcue.

Some Related Papers:

[1] Azcue and Muler (2015). Optimal dividend payment and regime switching in a compound Poisson risk model. SIAM J. Control Optim., 53, 3270–3298.

[2] Budhiraja and Ross (2007). Convergent numerical scheme for singular stochastic control with state constraints in a portfolio selection problem. SIAM J. Control Optim., 45, 2169-2206.

[3] Souganidis (1985). Approximation schemes for viscosity solutions of Hamilton–Jacobi equations, J. Differential Equations, 56 , pp. 345–390.