Nonlocal Perimeter, Curvature and Minimal Surfaces for measurable sets

JOSÉ M. MAZÓN

Departamento de Análisis Matemático
Universitat de Valencia

Abstract:

Our aim in this talk is to study the nonlocal perimeter associated with a nonnegative radial kernel $J : \mathbb{R}^N \to \mathbb{R}$, compactly supported, verifying $\int_{\mathbb{R}^N} J(z)dz = 1$. The nonlocal perimeter studied here is given by the interactions (measured in terms of the kernel J) of particles from the outside of a set with particles from the inside, that is,

$$P_J(E) := \int_E \left(\int_{\mathbb{R}^N \setminus E} J(x-y)dy \right) dx.$$

We prove that when the kernel J is appropriately rescaled, the nonlocal perimeter converges to the classical local perimeter. Associated with the kernel J and the previous definition of perimeter we can consider minimal surfaces. In connexion with minimal surfaces we introduce the concept of J-mean curvature at a point x, that we denote by $H^J_{\partial E}(x)$, and we show that again under rescaling we can recover the usual notion of mean curvature. In addition, we study the analogous to a Cheeger set in this nonlocal context and show that a set Ω is J-calibrable (Ω is a J-Cheeger set of itself) if and only if there exists τ such that $\tau(x) = 1$ if $x \in \Omega$ satisfying $-\lambda^J_\Omega \tau \in \Delta^J_1 \chi_\Omega$, here λ^J_Ω is the J-Cheeger constant $\lambda^J_\Omega = \frac{P_J(\Omega)}{|\Omega|}$ and, Δ^J_1 is given, formally, by

$$\Delta^J_1 u(x) = \int_{\mathbb{R}^N} J(x-y) \frac{u(y)-u(x)}{|u(y)-u(x)|} dy.$$

Moreover, we also provide a different characterization of calibrable sets using the nonlocal J-mean curvature.

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