Regularity theory for nonlocal filtration equations

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Abstract

We study the nonlinear and nonlocal Cauchy problem

$$\partial_t u + \mathcal{L}\varphi(u) = 0 \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+, \qquad u(\cdot, 0) = u_0,$$

where $\mathcal{L}$ is a Lévy-type nonlocal operator with a kernel having a singularity at the origin as that of the fractional Laplacian, but can be very irregular. The nonlinearity φ is nondecreasing and continuous, and the initial datum u_0 is assumed to be in $L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ and may change sign. We prove existence and uniqueness of a bounded weak solution, which is continuous assuming φ is neither too singular nor too degenerate. For a wide class of nonlinearities, including the porous media case and the fast diffusion case, $\varphi(u) = |u|^{m-1}u$, $0 < m < \infty$, these solutions turn out to be Hölder continuous at every point for $t > 0$. They are moreover classical if $\mathcal{L} = (-\Delta)^{\sigma/2}$, $0 < \sigma < 2$, provided $u_0 \geq 0$. We also obtain further regularity, even C^∞, when φ is smooth and nondegenerate, $\varphi' > 0$.

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