SYMMETRY BREAKING FOR AN ELLIPTIC EQUATION INVOLVING THE FRACTIONAL LAPLACIAN

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We study the symmetry breaking phenomenon for an elliptic equation involving the fractional Laplacian in a large ball. More precisely, we prove the following theorem, which is the analogue of a result of P. Sintzoff in [1], for the local case $s = 1$:

Theorem 1. *Let* $n \geq 2$, $1/2 < s < 1$, $2 < p < 2^* = \frac{2n}{n-2s}$, $0 < a < n$ *and* $b > \frac{ap}{2}$. *If in addition,*

$$a(p - 2 - 2ps) + 4bs < 2s(p - 2)(n - 1) \tag{1}$$

Then for every $R > 0$ *large enough, problem*

$$\begin{cases} (-\Delta)^s u + |x|^a u = |x|^b u^{p-1} & \text{in } B_R \\ u > 0 \text{ a.e. in } B_R, \quad u \equiv 0 \text{ in } \mathbb{R}^n - B_R \end{cases} . \tag{2}$$

has a nontrivial radial weak solution and a nonradial one (in the natural energy space for this problem) .

The argument is based on a comparison of the energy levels between the associated Rayleigh quotients for radial and non radial functions as $R \to \infty$.

Our main tool is an extension of the Strauss radial lemma involving the fractional Laplacian, which might be of independent interest (See also [3] for a detailed discusion of this kind of inequalities). From this inequality, we derive compact embedding theorems for a Sobolev space of radial functions with power weights.

References

[1] P. Sintzoff, *Symmetry and singularities for some semilinear elliptic problems*. Ph.D. Thesis Université Catolique de Louvain, (2005). The results have been published in *Symmetry of solutions of a semilinear elliptic equation with unbounded coefficients* Differential Integral Equations Volume 16, Number 7 (2003), 769–786.

[2] P. De Nápoli. *Symmetry breaking for an elliptic equation involving the Fractional Laplacian*. To appear in Differential and Integral Equations. Preprint `https://arxiv.org/abs/1409.7421`.

[3] P. L. De Nápoli and I. Drelichman. *Elementary proofs of embedding theorems for potential spaces of radial functions*. In *Methods of Fourier Analysis and Approximation Theory* (pp. 115-138). Springer International Publishing.

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