Blow-up of solutions of semilinear heat equations at the almost Hénon critical exponent

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Abstract

We study the problem

$$\begin{cases} u_t - \Delta u = |x|^{\alpha}|u|^{\frac{4+2\alpha}{N-2}-\varepsilon}u & \text{in } B_1 \times (0,\infty) \\ u = 0 & \text{on } \partial B_1 \times (0,\infty) \\ u = u_0 & \text{in } B_1 \times \{0\}, \end{cases} \tag{P_ε}$$

where B_1 is the unit ball in $\mathbb{R}^N$, $N > 2$, $\varepsilon > 0$ is an small parameter, and $\alpha > 0$ is a real number which is not an even integer. We show that if $\varepsilon > 0$ is small enough, then there exists a sign-changing stationary solution ψ_ε of (P_ε) such that the solution of (P_ε) with initial value $u_0 = \lambda\psi_\varepsilon$ blows up in finite time if $|\lambda - 1| > 0$ is sufficiently small.

References

[1] S. Alarcón, *Multiple sign-changing solutions at the almost Hénon critical exponent*, preprint.

[2] V. Marino, F. Pacella, B. Sciunzi, *Blow up of solutions of semilinear heat equations in general domains*, Comm. Contemp. Math. **17** (2015) 1350004, 17 pp.

[3] T. Cazenave, F. Dickstein, F.B. Weissler, *Sign-changing stationary solutions and blowup for the nonlinear heat equation in a ball*, Math. Ann. 344 (2009) 431-449.

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