The Linearization of the Schrödinger Equation

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Abstract

A new foundation to Hamiltonian Analytical Mechanics, named two fold or alternative Hamiltonization, furnishes two Hamiltonian functions, a linear in the momenta and the usual one. As one of the consequences of this Hamiltonization procedure in quantum mechanics there is the possibility of the linearization of the Schrödinger equation.

In this procedure the Hamiltonian function must be a solution to the PDE obtained by the substitution of the first set of Hamiltonian canonical equations of motion in the Hamilton´s definition of a Hamiltonian function. The canonical momenta are provided by second set of Hamiltonian canonical equations of motion.

The main change proposed in this procedure is that the conjugate momenta should not be postulated a priori, but instead of this, they are determinate as a consequence of a canonical description of the mechanical system. It is also proved that Hamilton's definition of the conjugate momenta is obtained by the imposition of the envelope condition in the solution of the PDE that defines the Hamiltonian function. Therefore in the singular mechanics the usual definition cannot be used as the PDE is linear in the momenta. It must be noted that the Hamiltonian yielded is identical to that obtained by Dirac, but with an additional advantage as there is no constraints, no need of new definitions as "weak equalities" or "super phase space"' nor a new variational procedure. Then this procedure can be applied to Singular (Dirac), Nambu, or Non Holonomic Mechanics, and can be used to the linearization of the Hamilton-Jacobi equation or to the determinate constants of motion.

The linearization of the Schrödinger equation can also be obtained from the Hamilton Jacobi one.

The same idea was extended for field theories singular or not. The usual Hamiltonian density and the momentum density are recovered by the envelope condition whenever it exists. In the singular case the result obtained is the same as in the Dirac theory and has the same advantage of no constraints. Therefore the usual quantization can be performed.

A generalization of the above procedure, named direct Hamiltonization, allows the determination of the Hamiltonian function for any mechanical system described or not by a Lagrangian. It was induced by the Hamiltonization of non holonomic systems where the Hamiltonian function is obtained from the system composed by the PDE that defines the Hamiltonian and the PDE given by the constraints in which is used the first set of the canonical equations of motion. As in this case the real Lagrangian is not known. In this approach we prove that any function of the generalized coordinates, generalized velocities and time can be added with no consequence on the description, generating an equivalent Hamiltonian. The difference is in the momentum defined by the condition of a canonical description of the system. As the direct Hamiltonization contains the alternative one, then the usual Hamiltonization and momenta is recovered while the envelope solution is selected after adding the Lagrangian function. Also this procedure assures the existence of a Hamiltonian function without any constraints whatsoever mechanical system is considered, therefore the usual quantization is always allowed. It can be expanded to field theory giving a direct Hamiltonization with the same consequences.

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