Energy cascade in turbulent flows using the sabra shell model.

Elder Joel Varas Pérez
evaras@unitru.edu.pe

Department of Mathematics, National University of Trujillo, Perú

Alexis Rodriguez Carranza
arodriguezca@unitru.edu.pe

Department of Mathematics, National University of Trujillo, Perú

Abstract

Transfer of energy from large to small scales in turbulent flows is described as a flux of energy from small wave numbers to large wave numbers in the spectral representation of the Navier-Stokes equation

$$\partial_t u_i(\mathbf{k}) = -\iota k_j \int (\delta_{i\ell} - \frac{k_i k'_\ell}{k^2}) u_j(\mathbf{k}') u_\ell(\mathbf{k}-\mathbf{k}')\mathrm{d}\mathbf{k}' - \nu k^2 u_i(\mathbf{k}) + f_i(\mathbf{k}) \qquad (1)$$

where ν is the viscosity and f_n is the external force. The problem of resolving the relevant scales in the flow corresponds in the spectral representation to determining the spectral truncation at large wave numbers. The effective number of degrees of freedom in the flow depends on the Reynolds number. The Kolmogorov scale η depends on Reynolds number as $\eta \sim Re^{-3/4}$, so the number of waves N necessary to resolve scales larger than η grows with Re as $N \sim \eta^{-3} \sim Re^{9/4}$. This means that even for moderate Reynolds numbers ~ 1000, the effective number of degrees of freedom is of the order of 10^7. A numerical simulation of the Navier Stokes equation for high Reynolds numbers is therefore impractical without some sort of reduction of the number of degrees of freedom.

Shell models of turbulence were introduced by Obukhov (1971) and Gledzer (1973). They consist of a set of ordinary differential equations structurally similar to the spectral Navier-Stokes equation (2). These models are much simpler and numerically easier to investigate than the Navier Stokes equation. For these models a scaling theory identical to the Kolmogorov theory has been developed, and they show the same kind of deviation from the Kolmogorov scaling as real turbulent systems do. Understanding the behavior of shell models in their own right might be a key for understanding the systems governed by the Navier Stokes equation. The shell models are constructed to obey the same conservation laws and symmetries as the Navier Stokes equation.

In this work we present the Sabra shell model

$$\dot{u}_n = \iota[k_n u^*_{n+1} u_{n+2} - \epsilon k_{n-1} u^*_{n-1} u_{n+1} + (1-\epsilon) k_{n-2} u_{n-2} u_{n-1}] - \nu k_n^2 u_n + f_n \quad (2)$$

for the study of the energy cascade of turbulence and we will show numerically that the energy dissipation is approximately -1/3 which is in agreement with the theory K41.

References

[1] Peter D. Ditlevsen , *Turbulence and Shell Models*, Cambridge University Press 2011.

[2] Stephen B. Pope, *Turbulent Flows.* Cambridge University Press, 2000.

[3] G. K. Batchelor, *An Introduction to Fluid Dynamics.* Cambridge Mathematical Library, 1967.