Nonlinear PDEs in applied electromagnetism

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Applied electromagnetism is an inexhaustible source of linear and nonlinear PDE problems. We will focus here on problems associated with the non-destructive evaluation of ferromagnetic materials. Typical applications are relatively low frequency (which means that the full hyperbolic Maxwell equations system can be simplified) and are usually well described by the so-called induced currents approximation (parabolic/elliptic system). To be specific, we can consider the equation

$$\sigma \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \nabla \times \boldsymbol{H}$$

where σ is the electrical conductivity (positive on conductors, zero in insulators, air, etc.), and vectors $\boldsymbol{B}$, $\boldsymbol{H}$ are the divergence free magnetic induction and the magnetic field, respectively.

For non-ferromagnetic materials $\boldsymbol{B}=\mu_0 \boldsymbol{H}$, with μ_0 a positive constant (the vacuum magnetic permeability), and the equation above is linear. Instead, for ferromagnetic materials the relationship between $\boldsymbol{B}$ and $\boldsymbol{H}$ is extremely complex, which gives rise to many different approximation ideas. For instance, it can be modelled as a vector multibranch nonlinearity (where the actual $\boldsymbol{B}(\boldsymbol{H})$ value depends both on past history and on the sign of the variation), as a nonlinear discontinuous function (thus giving rise to free boundary problems), as a nonlinear smooth function $\boldsymbol{B}=\mu(||\boldsymbol{H}||)\,\boldsymbol{H}$ (where $\mu(||\boldsymbol{H}||)$ is the magnetic permeability of the material, usually much larger than μ_0), etc.

Even in this last very simplified form, permeability can be given many different parameterizations, like

$$\mu - \mu_0 = \frac{1}{a\|\boldsymbol{H}\| + b} \ .$$

In this poster we present several nonlinear elliptic/parabolic problems of interest in applications, which may lead to free boundary problems according to modeling choices, with the goal of fostering interest on applied problems, and help illuminating from the theory some of the very complex issues involved in the techniques.