Necessary and sufficient conditions for the existence of positive (c, ω)-periodic solutions of a Nicholson type delay system

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In this work we present, by means of topological degree methods, necessary and sufficient conditions for the existence of at least one positive (c, ω)-periodic solution of a Nicholson type delay system

$$\begin{cases} x_1'(t) = -\delta_1(t)x_1(t) + \beta_1(t)x_2(t) + p_1(t)x_1(t-\tau_1)e^{-a_1(t)x_1(t-\tau_1)} \\ x_2'(t) = -\delta_2 t)x_2(t) + \beta_2(t)x_1(t) + p_2(t)x_2(t-\tau_2)e^{-a_2(t)x_2(t-\tau_2)} \end{cases} \quad (1)$$

where $\delta_i, \beta_i, p_i \in C(\mathbb{R}, \mathbb{R}^+)$ are ω-periodic, $a_i \in C(\mathbb{R}, \mathbb{R}^+)$ are $(1/c, w)$-periodic for $i = 1, 2$ and τ_i are positive constants.

The class of (c, ω) functions includes periodic, anti-periodic, Bloch functions but also unbounded functions when $|c| \neq 1$. More specifically, for a given pair (c, ω) such that $c \in \mathbb{C} \backslash \{0\}, \omega > 0$, we shall say that $g \in C(\mathbb{R}, \mathbb{C})$ is a (c, ω)-periodic function if $g(t + \omega) = cg(t)$ for all $t \in \mathbb{R}$ (see e.g. [2], [3]).

Systems of type (1) were used, for example, in marine protected areas and to describe the dynamics of the B-cells of the lymphocytic leukemia ([1],[4]).

References

[1] L. Berezansky, L. Idels, L. Troib. Global dynamics of Nicholson-type delay system with applications. Nonlinear Anal. Real World Appl. 12 (2011), 436–445.

[2] M. Pinto. Ergodicity and Oscillations. Conference in Universidad Católica del Norte. 2014.

[3] M. Pinto. Pseudo-almost periodic solutions of neutral integral and differential equations with applications. Nonlinear Anal. 72(12)(2010), 4377–4383.

[4] H. Smith. An Introduction to Delay Differential Equations with Applications to the Life Sciences, Springer-Verlag, New York, 2011.