A priori estimates for viscosity solutions of fully nonlinear parabolic equations

G. C. Rampasso
University of Campinas - UNICAMP, Brazil.

girampasso@ime.unicamp.br

Abstract

Since the foundation of the modern analysis of partial differential equations, the regularity theory for solutions has been one of the main subjects of research. In particular, a substantial class of equations involve operators whose ellipticity degenerates along an *a priori* unknown region: the free boundary. Since this region can also vary according to the solution itself, regularity become a delicate issue depending on the diffusion control near such a region.

In special, we are interested in the study of fully nonlinear parabolic equation with strong absorption and gradient dependence, as follows

$$\begin{cases} \partial_t u - F(D^2(u)) = -\gamma u^{\gamma-1}\Gamma(|\nabla u|)\chi_{\{u>0\}}, & \text{in} \quad Q_1 = B_1(0)\times(-1,0] \subset \mathbb{R}^{d+1} \\ u(x,t) = 1, \quad \text{on} \;\; \partial B_1(0)\times(-1,0], \\ u(x,-1) = u_0(x) \;\; \text{in} \;\; B_1(0) \end{cases} \tag{PDE}$$

where $F\colon \mathcal{S}(d) \to \mathbb{R}$ is (λ,Λ)-elliptic, and convex, $\mathcal{S}(d)$ is the set of all symmetric $d\times d$ real matrix, $1<\gamma<2$, and $\Gamma\colon [0,\infty)\to\mathbb{R}$ is defined by $\Gamma(t) = 1+t^m$, $0<m<2-\gamma$.

The purpose of this work is to obtain *a priori* estimates for viscosity solutions, that are, in some sense, uniform with respect to some analytical properties of the right-hand side of (PDE). Hence, we generalize some recent results of Montenegro-Queiroz-Teixeira and Choe-Weiss.

This is part of a work joint Olivaine S. de Queiroz.