Mathematical model for acid water neutralization with anomalous and fast diffusion

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Abstract

In this contributed talk we model the neutralization of an acid solution in which the hydrogen ions are transported according to Cattaneo's diffusion. The latter is a modification of classical Fickian diffusion in which the flux adjusts to the gradient with a positive relaxation time. Accordingly the evolution of the ions concentration is governed by the hyperbolic telegraph equation instead of the classical heat equation. We focus on the specific case of a marble slab reacting with a sulphuric acid solution and we consider a one-dimensional geometry. We show that the problem is multi-scale in time, with a reaction time scale that is larger than the diffusive time scale, so that the governing equation is reduced to the one-dimensional wave equation. The mathematical problem turns out to be a hyperbolic free boundary problem where the consumption of the slab is described by a nonlinear differential equation. Global well posedness is proved and some numerical simulations are provided.

Keywords: neutralization, reaction kinetics, multi-scale modeling, free boundary problem, anomalous diffusion

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