A necessary condition for strong hyperbolicity of general first order systems.

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We study strong hyperbolicity of first order partial differential equations for systems with differential constraints. In those cases, the number of equations is larger than the unknown fields, therefore, the standard hyperbolicity does not directly apply. To deal with this problem one introduces a new tensor, called a hyperbolizer, which selects a subset of equations so that they might be used as evolution equations for the unknowns. Different hyperbolizers, may or may not lead to strongly hyperbolic evolution systems.

To sort-out this issue, we look for a condition which is independent of the hyperbolizers but which, if satisfied, implies the non-existence of strongly hyperbolic reductions [1]. We look at the singular value decomposition of the whole system and show that if the principal symbol is appropriately perturbed (via a parameter ε), we obtain perturbed characteristic surfaces with given perturbed singular values. If any of the singular values is order $O\left(\varepsilon^{l}\right)$, with $l \geq 2$, hyperbolicity breaks down. In addition, we notice that the calculations at orders $O\left(\varepsilon^{0}\right)$ and $O\left(\varepsilon^{1}\right)$ can be done in a covariant way.

These results are reached by connecting the perturbation of singular values with the diagonalization of square matrices, leading to a different and simpler proof of Kreiss's matrix theorem. We then extend these conclusions to systems with constraints.

Finally we apply the results to examples in physics, such as Force-Free Electrodynamics in Euler potentials form and charged fluids with finite conductivity. We find that they are only weakly hyperbolic.

References

[1] Abalos, Fernando. "A necessary condition for strong hyperbolicity of general first order systems." arXiv preprint arXiv:1707.05011 (2017).