V INTERNATIONAL SYMPOSIUM
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Book of abstracts

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Models of Segregation

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Abstract

Mathematically, segregation describes the configuration of several different species that diffuse but are penalized for overlapping This involves many phenomena beyond segregation of species: Particle annihilation, harmonic maps into singular manifolds, phase transitions.

We will discus recent work, one involving fully non linear diffusion, related to optimal control, the other on the case of local-non local interaction when a species propagates continuously and the other through Levy jumps

Near field asymptotics for the porous medium equation in exterior domains

Carmen Cortázar

Pontifical Catholic University of Chile

Abstract

Let $\mathcal{H} \subset \mathbb{R}^N$ be a non-empty bounded open set. We consider the porous medium equation in the complement of $\mathcal{H}$, with zero Dirichlet data on its boundary and nonnegative compactly supported integrable initial data.

Kamin and Vázquez, in 1991, studied the large time behavior of solutions of such problem in space dimension 1. Gilding and Goncerzewicz, in 2007, studied this same problem dimension 2. Using their results in the outer field we study the large time behavior of the solution in the near field scale, in particular in bounded sets of $\mathbb{R}^N \setminus \mathcal{H}$.

This a joint work with Fernando Quirós (Universidad Autonoma de Madrid, Spain) and Noem Wolanski (Universidad de Buenos Aires, Argentina).

Fractional order Orlicz-Sobolev spaces and applications

Julián Fernández Bonder

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Abstract

In this talk I will give what we believe is the *natural* definition of fractional order Orlicz-Sobolev spaces. After showing that this spaces are well-behaved from a functional analysis point of view, we investigate the limit as the fractional parameter $s \uparrow 1$. Then we apply these results to the study of some fractional non-standard growth elliptic-type problems.

This work is in collaboration with Ariel Salort from Universidad de Buenos Aires.

Non conservative Boltzmann type equation for Bose-Einstein Condensates for Cold Bosons and Wave Turbulence models

Irene Martinez Gamba

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Abstract

We will discuss the similarities and differences of these two models for Bose-Einstein Condensates for Cold Bosons and Wave Turbulence models for gravity driven stratified flows. These are models of evolution for probability density distributions, and can be viewed as aggregation models that do not conserve mass.

We will describe their differences in terms of existence and uniqueness and long time behavior. This theory follows from solving these two flows models of non-linear no-local integral differential equations in Banach spaces.

Multiplicity results for some quasilinear elliptic problems

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Abstract

Our aim in this talk is to study the nonlocal perimeter associated with a nonnegative radial kernel $J : \mathbb{R}^N \to \mathbb{R}$, compactly supported, verifying $\int_{\mathbb{R}^N} J(z)dz = 1$. The nonlocal perimeter studied here is given by the interactions (measured in terms of the kernel J) of particles from the outside of a set with particles from the inside, that is,

$$P_J(E) := \int_E \left(\int_{\mathbb{R}^N \setminus E} J(x-y)dy \right) dx.$$

We prove that when the kernel J is appropriately rescaled, the nonlocal perimeter converges to the classical local perimeter. Associated with the kernel J and the previous definition of perimeter we can consider minimal surfaces. In connexion with minimal surfaces we introduce the concept of J-mean curvature at a point x, that we denote by $H^J_{\partial E}(x)$, and we show that again under rescaling we can recover the usual notion of mean curvature. In addition, we study the analogous to a Cheeger set in this nonlocal context and show that a set Ω is J-calibrable (Ω is a J-Cheeger set of itself) if and only if there exists τ such that $\tau(x) = 1$ if $x \in \Omega$ satisfying $-\lambda^J_\Omega \tau \in \Delta^J_1 \chi_\Omega$, here λ^J_Ω is the J-Cheeger constant $\lambda^J_\Omega = \frac{P_J(\Omega)}{|\Omega|}$ and, Δ^J_1 is given, formally, by

$$\Delta^J_1 u(x) = \int_{\mathbb{R}^N} J(x-y) \frac{u(y)-u(x)}{|u(y)-u(x)|} dy.$$

Moreover, we also provide a different characterization of calibrable sets using the nonlocal J-mean curvature.

Joint work with Julio Rossi and Julián Toledo

Up to the boundary gradient estimates in nonlinear free boundary problems

Diego Moreira

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Abstract

In this talk, we present some ingredients that lead to gradient estimates up to the boundary in nonlinear free boundary problems. As an application and main motivation, we apply these estimates for singular perturbation problems of flame propagation type.

Logarithmic corrections in Fisher-KPP problems for the Porous Medium Equation

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Abstract

We consider the large time behaviour of solutions to the Porous Medium Equation with a Fisher-KPP type reaction term

$$u_t = \Delta u^m + u - u^2 \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+, \qquad u(\cdot, 0) = u_0 \quad \text{in } \mathbb{R}^N,$$

$m > 1$, for nonnegative, nontrivial, radially symmetric, bounded and compactly supported initial data u_0. It is well known that in spatial dimension one there is a minimal speed $c_* > 0$ for which the equation admits a traveling wave solution Φ_{c_*} with a finite front. We prove that there exists a second constant $c^* > 0$ independent of the dimension N and the initial function u_0, such that

$$\lim_{t\to\infty} \left\{ \sup_{x\in\mathbb{R}^N} |u(x,t) - \Phi_{c_*}(|x| - c_* t + (N-1)c^* \log t - r_0)| \right\} = 0$$

for some $r_0 \in \mathbb{R}$ (depending on u_0). Moreover, the radius, $h(t)$, of the support of the solution at time t satisfies

$$\lim_{t\to\infty} \left[h(t) - c_* t + (N-1)c^* \log t \right] = r_0.$$

Thus, in contrast with the semilinear case $m = 1$, we have a logarithmic correction only for $N > 1$.

Joint work with Yihong Du and Maolin Zhou.

Games for PDEs with eigenvalues of the Hessian

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Abstract

For a function $u : \Omega \subset \mathbb{R}^N \mapsto \mathbb{R}$, we consider the Hessian, $D^2 u$, and its ordered eigenvalues

$$\lambda_1(D^2 u) \leq \ldots \leq \lambda_N(D^2 u).$$

Here our main concern is the Dirichlet problems for the equations:

$$P_k^+(D^2 u) := \sum_{i=N-k+1}^{N} \lambda_i(D^2 u) = 0, \tag{1}$$

(note that P_k^+ is just the sum of the k largest eigenvalues)

$$P_k^-(D^2u) := \sum_{i=1}^{k} \lambda_i(D^2u) = 0, \tag{2}$$

(P_k^- is the sum of the k smallest eigenvalues) and, more generally, any sum of k different eigenvalues

$$P_{i_1,\dots,i_k}(D^2u) := \sum_{i_1,\dots,i_k} \lambda_{i_j}(D^2u) = 0. \tag{3}$$

These operators appear in connection with geometry but our goal is to provide a probabilistic interpretation.

We will describe games whose values approximate viscosity solutions to these equations in the same spirit as the random walk can be used to approximate harmonic functions.

Joint work with P. Blanc (U. Buenos Aires, Argentina).

Multi-Junction and corners in FB problems

Henrik Shahgholian

The Royal Institute of Technology, Stockholm

Abstract

I shall discuss formation of multi-junction points for FB problems, as well as the behavior of FB close to corner points of fixed boundaries.

Free boundary problems in complex materials

Eduardo Teixeira

University of Central Florida

Abstract

In mathematical models for diffusion, leading (2nd order) coefficients often convey the tangible properties of the medium in which the phenomena take place. In turn, physical complexity of the model is largely encoded within the structure of the media. In this talk,I will discuss some recent investigations on free boundary problems modeled in complex media.

An Aubin-Lions type compactness result

Joana Terra

FAMAF–UNC–ARGENTINA

Abstract

In this talk I will show an Aubin-Lions type compactness result for time-evolving Bochner spaces. Such spaces appear as the correct functional setting where to search for solutions to partial differential equations considered in time- evolving domains. This is a joint work with A. Alphonse and C. Elliott.

Blow-up of solutions of semilinear heat equations at the almost Hénon critical exponent

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Abstract

We study the problem

$$\begin{cases} u_t - \Delta u = |x|^{\alpha}|u|^{\frac{4+2\alpha}{N-2}-\varepsilon}u & \text{in } B_1 \times (0,\infty) \\ u = 0 & \text{on } \partial B_1 \times (0,\infty) \\ u = u_0 & \text{in } B_1 \times \{0\}, \end{cases} \tag{P_ε}$$

where B_1 is the unit ball in $\mathbb{R}^N$, $N > 2$, $\varepsilon > 0$ is an small parameter, and $\alpha > 0$ is a real number which is not an even integer. We show that if $\varepsilon > 0$ is small enough, then there exists a sign-changing stationary solution ψ_ε of (P_ε) such that the solution of (P_ε) with initial value $u_0 = \lambda\psi_\varepsilon$ blows up in finite time if $|\lambda - 1| > 0$ is sufficiently small.

Symmetry breaking for an elliptic equation involving the Fractional Laplacian

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Abstract

We study the symmetry breaking phenomenon for an elliptic equation involving the fractional Laplacian in a large ball. More precisely, we prove the following theorem, which is the analogue of a result of P. Sintzoff in [1], for the local case $s = 1$:

Theorem 1. Let $n \geq 2$, $1/2 < s < 1$, $2 < p < 2^* = \frac{2n}{n-2s}$, $0 < a < n$ and $b > \frac{ap}{2}$. If in addition,

$$a(p - 2 - 2ps) + 4bs < 2s(p - 2)(n - 1) \tag{4}$$

Then for every $R > 0$ large enough, problem

$$\begin{cases} -(-\Delta)^s u + |x|^a u = |x|^b u^{p-1} & \text{in } B_R \\ u > 0 \text{ a.e. in } B_R, \quad u \equiv 0 \text{ in } \mathbb{R}^n - B_R \end{cases}. \tag{5}$$

has a nontrivial radial weak solution and a nonradial one (in the natural energy space for this problem) .

The argument is based on a comparison of the energy levels between the associated Rayleigh quotients for radial and non radial functions as $R \to \infty$.

Our main tool is an extension of the Strauss radial lemma involving the fractional Laplacian, which might be of independent interest (See also [2] for a detailed discusion of this kind of inequalities). From this inequality, we derive compact embedding theorems for a Sobolev space of radial functions with power weights.

Reference

1. P. Sintzoff. *Symmetry and singularities for some semilinear elliptic problems*. Ph.D. Thesis Universit Catolique de Louvain, (2005). The results have been published in *Symmetry of solutions of a semilinear elliptic equation with unbounded coefficients* Differential Integral Equations Volume 16, Number 7 (2003), 769–786.
2. P. L. De Nápoli and I. Drelichman. *Elementary proofs of embedding theorems for potential spaces of radial functions*. In *Methods of Fourier Analysis and Approximation Theory* (pp. 115-138). Springer International Publishing.

Regularity theory for nonlocal filtration equations

Arturo de Pablo

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Abstract

We study the nonlinear and nonlocal Cauchy problem

$$\partial_t u + \mathcal{L}\varphi(u) = 0 \quad \text{in } \mathbb{R}^N \times \mathbb{R}_+, \quad u(\cdot, 0) = u_0,$$

where $\mathcal{L}$ is a Lévy-type nonlocal operator with a kernel having a singularity at the origin as that of the fractional Laplacian, but can be very irregular. The nonlinearity ′ is nondecreasing and continuous, and the initial datum u_0 is assumed to be in $L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ and may change sign. We prove existence and uniqueness of a bounded weak solution, which is continuous assuming φ is neither too singular nor too degenerate. For a wide class of nonlinearities, including the porous media case and the fast diffusion case, $\varphi(u) = |u|^{m-1}u$, $0 < m < 1$, these solutions turn out to be Hölder continuous at every point for $t > 0$. They are moreover classical if $\mathcal{L} = (-\Delta)^{\sigma/2}, 0 < \sigma < 2$, provided $u_0 \geq 0$. We also obtain further regularity, even C^∞, when φ is smooth and nondegenerate, $\varphi' > 0$.

Joint works with F. Quirós and A. Rodríguez.

H^2 Regularity for the $p(x)$–Laplacian in two-dimensional convex domains

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Abstract

In an image processing problem, the aim is to recover the real image I from an observed image ξ of the form $\xi = I + \eta$, where η is a noise. In [1], the authors introduce a model that involves the $p(x)$–Laplacian (i.e $\Delta_{p(x)} u = \operatorname{div}(|\nabla u|^{p(x)-2} \nabla u)$), for some function $p : \Omega \to [p_1, 2]$, with $p_1 > 1$. More precisely, they minimize in $W^{1,p(\cdot)}(\Omega) \cap L^2(\Omega)$ the functional

$$\int_\Omega |\nabla u|^{p(x)} + \frac{\lambda}{2} \int_\Omega |u - \xi|^2 \, dx,$$

where λ is a parameter and the function $p(x)$ encodes the information on the regions where the gradient is sufficiently large (at edges) and where the gradient is close to zero (in homogeneous regions). In this manner, the model avoids the *staircasing* effect still preserving the edges.

Motivated by this application and to its applications to prove the rate of convergence for the associated continuous Galerkin Finite Element Method in the two dimensional case, we study some Sobolev regularity results of the solutions.

In particular, in this talk we disuse how we can prove the H^2 regularity of the solutions in the two dimensional case, when the domain is regular or convex.

This is a joint work with L. M. Del Pezzo.

Reference

1. Yunmei Chen, Stacey Levine, and Murali Rao. *Variable exponent, linear growth functionals in image restoration*, SIAM J. Appl. Math. **66**, no. 4, (2006) 1383–1406 (electronic).

Multidimensional optimization of dividends: a problem with free regions

Nora Muller

Univ. Torcuado Di Tella

Abstract

In [1], we considered the problem of maximizing the expectation of discounted dividends until a stopping time. This stopping time is defined as the minimum between the ruin time (that is when the reserve of the company becomes negative) and a decision time that stops the process (with an extra dividend payment at that time which depends on the reserve by a function f) . The uncontrolled reserve follows a simple stochastic jump process (compound Poisson with negative jumps). This is a mixed singular control/optimal stopping problem. In the present work, we generalize [1] to a multidimensional setting (n associated companies). As in the unidimensional case, it can be proved that the optimal value function V can be characterized as the smallest viscosity supersolution of the associated non-linear integro-differential Hamilton-Jacobi-Equation (it is also a viscosity solution of this equation).The HJB equation is a maximum between $n+2$ operators equal to zero. The $n+2$ regions of the state space $\mathbb{R}_+^n$ in which each operator applied to V is zero determine the optimal strategy: One operators is $V-f$ and so f can be regarded as an obstacle (the corresponding set is associated to the decision region), another operator is an integro-differential one (arising from the jump process) and the corresponding set gives the non-action region, the other n operators are related to the regions in which each company pays dividends. The hardest part of the problem is to find these $n+2$ regions. The main contribution of this work is to approximate these regions and V by a numerical procedure; we propose to consider a family of admissible strategies (based on the $n+2$ operators of the HJB equation) in a suitable grid that satisfy a discrete version of the HJB equation and show that the value function of these strategies converge locally uniformly to the optimal value function V as the mesh size goes to zero. We also show examples.

This is a joint work with Pablo Azcue.

Some Related Papers:

[1] Azcue and Muler (2015). Optimal dividend payment and regime switching in a compound Poisson risk model. SIAM J. Control Optim., 53, 3270–3298.

[2] Budhiraja and Ross (2007). Convergent numerical scheme for singular stochastic control with state constraints in a portfolio selection problem. SIAM J. Control Optim., 45, 2169–2206.

[3] Souganidis (1985). Approximation schemes for viscosity solutions of Hamilton – Jacobi equations, J. Differential Equations, 56 , pp. 345–390.

Hölder-regularity for asymptotically elliptic operators

Edgard A. Pimentel

PUC–Rio

Abstract

In this talk we address fully nonlinear equations driven by asymptotically elliptic operators. We prove local estimates for the solutions in Hölder. In particular, we establish $C^{1,Log-Lip}$–estimates. Our arguments rely on a geometric double blow-up argument. First, we approximate the original problem by a uniformly elliptic one. Then, we displace our assumptions to the recession pro

le of the latter. We discuss consequences of our main result to the regularity theory of important examples. Namely, the Monge-Ampère equation and the truncated Laplacian operators.

On fractional Cheeger's inequality

Ariel Salort

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Abstract

For open bounded sets, we prove that the first eigenvalue of the fractional p–Laplacian of order s is estimated from below by the fractional Cheeger constant of order s. The constant entering in the estimate displays the correct asymptotic behaviour as the fractional order of differentiability s goes to 0 and 1.

The singular free boundary in the Signorini problem

Mariana Smit Vega Garcia

University of Washington

Abstract

In this talk I will overview the Signorini problem for a divergence form elliptic operator with Lipschitz coefficients, and I will describe a few methods used to tackle two fundamental questions: what is the optimal regularity of the solution, and what can be said about the singular free boundary. The proofs are based on Weiss and Monneau type monotonicity formulas. This is joint work with Nicola Garofalo and Arshak Petrosyan.

A Non-classical Heat Conduction Problem with a Source Depending of the Total Heat Flux on the Boundary

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Abstract

Motivated by the modeling of the temperature regulation within a medium we consider the non-classical heat conduction equation in a semi-space n–dimensional domain $D = \mathbb{R}^+ \times \mathbb{R}^{n-1}$ for which the internal energy supply depends on the total heat flux in the time variable on the boundary $S = \partial D = \{0\} \times \mathbb{R}^{n-1}$, with homogeneous Dirichlet boundary condition and an initial condition. The problem consists in finding the temperature $u = u(x,t)$ such that the following conditions are satisfied:

$$\begin{cases} u_t - \Delta u = -F\left(\int_0^t u_x(0,y,s)\,ds\right), & x > 0,\ y \in \mathbb{R}^{n-1},\ t > 0 \\ u(0,y,t) = 0, & y \in \mathbb{R}^{n-1},\ t > 0 \\ u(x,y,0) = h(x,y), & x > 0,\ y \in \mathbb{R}^{n-1} \end{cases}$$

By using a Volterra integral equation of second kind in the time variable with a parameter $y \in \mathbb{R}^{n-1}$ the solution to this problem is obtained. The solution to that Volterra integral equation is the heat flux on S , which is an additional unknown of the considered nonlinear problem. We show that a unique local solution exists, which can be extended globally in time.

Finally, a one-dimensional case is studied and we obtain the explicit solution by using the Adomian method and we derive its properties. We must use a double induction principle in order to obtain that explicit solution which is also related to the Mittag-Leffler functions. Moreover, we obtain a relationship between this solution with a third order ordinary differential equation with a singular second member, with two initial conditions at the fixed boundary and an integral boundary condition within the domain.

Joint work with Mahdi Boukrouche from Lyon University

Free Transmission Problems

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Abstract

We study transmission problems with free interfaces from one random medium to another. Solutions are required to solve distinct partial differential equations, L_+ and L_-, within their positive and negative sets respectively. A corresponding flux balance from one phase to another is also imposed. We establish existence and L^∞ bounds of solutions. We also prove that variational solutions are non-degenerate and develop the regularity theory for solutions of such free boundary problems.

A limiting optimal design problem governed by non-local diffusion

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Abstract

We study a p–fractional optimal design problem under volume constraint taking special care of the case when p is large, obtaining in the limit profile a free boundary problem modelled by the Hölder Infinity Laplacian operator. A necessary and sufficient condition is imposed in order to obtain the uniqueness of solutions to the limiting problem, and, under such a condition, we find precisely the optimal configuration for the limit problem. We also prove the sharp $C^{0,s}_{loc}$ regularity for any limiting solution.

Non-linear homogenization. Modeling composites

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Abstract

The stresses that an ideally plastic material can withstand form a bounded closed set in the space of symmetric 3×3 real matrices with the standard distance. This set, which is a material property, is called the yield set. Unlike brittle materials, ideally plastic materials do not break. When subjected to a stress that is in the boundary of the yield set, the material experiences a permanent deformation, called plastic deformation.

Fiber reinforced composites are materials made of solid fibers embedded in a weaker solid referred to as the matrix. In this work, the fibers are assumed to be infinite in length and parallel. Thus, symmetry allows the study to be restricted to a plane perpendicular to the fibers.

The microstructure of the composite refers to the description of the re- gions in space occupied by the fibers and the matrix. It is described by a characteristic function χ, $\chi(x) = 1$ if x is inside a fiber and

$\chi(x) = 0$ otherwise. The microstructures are assumed to be periodic, i.e. χ is $[0,1]^2$−periodic. The consideration fiber reinforced composites where both the fibers and the matrix are made of ideally plastic solids with a different yield set leads to the following homogenization problem:

Let M represent the relative strength of the fibers with respect to the matrix. It is assumed that $M >> 1$. A two-dimensional vector field σ is said to be admissible if it is $[0,1]^2$−periodic, it satisfies the restrictions $\|(x)\| M\chi(x) + 1\chi(x)$ and the equilibrium equation $\nabla \cdot \sigma = 0$. The goal is to compute the set $Y_{hom} = \{\tau : \tau = \langle\rangle$, for some admissible vector field$\sigma\}$, where $\langle\rangle$ denotes the average of σ. A new and sharp bound on the set Y_{hom} is obtained in this work.

A principle of relatedness for systems with small delays

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Abstract

A general result by Krasnoselskii establishes that, if we consider a fixed point operator $K : U \subset C_T(\mathbb{R}, \mathbb{R}^N) \to C_T(\mathbb{R}, \mathbb{R}^N)$ associated to

$$u'(t) = g(t, u(t)), \quad u(0) = u(T),$$

and $P : G \subset \mathbb{R}^N \to \mathbb{R}^N$ is the Poincaré map, then, under appropriate hypotheses, the Leray-Schauder degree of $I - K$ in U coincides with the Brouwer degree of $I - P$ in G.

In this work, we extend this relatedness principle to a system of DDEs

$$u'(t) = g(u(t), u(t-\tau)) + p(t), \tag{6}$$

where $\tau > 0$, $g : \overline{\Omega} \times \overline{\Omega} \to \mathbb{R}^N$ is continuously differentiable and $\Omega \subset \mathbb{R}^N$. In this case the Poincaré map is defined in $C([-\tau, 0], \mathbb{R}^N)$. Based on the result for $\tau = 0$, we shall prove that the principle holds for small values of τ.

As a consequence, we deduce that, for nearly all, i. e. except a countable set, $T > 0$, if $G(u) := g(u, u)$ is an inward pointing field, then the system with $p = 0$ has an equilibrium $e \in \Omega$ and, furthermore, the index of the Poincaré map of the linearised system for $\tau = 0$ is equal to -1, then problem (6) has at least two (generically three) T−periodic solutions, provided that $p \in C(\mathbb{R}, \mathbb{R}^N)$ is T−periodic and close to the origin.

Moreover, extending another result by Krasnoselskii, we prove that the previous assumptions imply that the equilibrium is unstable.

Opinion formation models with heterogeneous persuasion and zealotry

Mayte Pérez–Llanos

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Abstract

In this work we propose an opinion formation model with heterogeneous agents. We assume that they have different power of persuasion, and each agent has its own level of zealotry, that is, an individual willingness to being convinced by other agent. Also, we include zealots or stubborn agents that never change opinions.

We derive a Bolzmann-like equation for the distribution of agents on the space of opinions, and we approximate it with a transport equation with a nonlocal drift term. We study the long-time asymptotic

behavior of solutions, characterizing the limit distribution of agents, which consists of the distribution of stubborn agents, plus a delta function at the mean of their opinions, weighted by they power of persuasion.

Moreover, we present explicit bounds on the rate of convergence, and we show that the time to convergence decreases when the number of stubborn agents increases. This is a striking fact observed in agent based simulations in different works.

Joint work with J. P. Pinasco (jpinasco@gmail.com) and Nicolás Saintier (nsaintie@dm.uba.ar), Universidad de Buenos Aires and CONICET, and Analía Silva (analia.silva82@gmail.com) Universidad Nacional de San Luis and CONICET.

Two different fractional Stefan problems which are convergent to the same classical Stefan problem

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Abstract

Two fractional Stefan problems are considered. The first one by using the Caputo derivative of order $\alpha \in (0,1)$:

$$\begin{array}{lll}
(i) & {}_0^C D_t^\alpha u(x,t) = \lambda^2 \frac{\partial}{\partial x^2} u(x,t), & 0 < x < s(t),\ 0 < t < T, \\
(ii) & u(x,0) = f(x), & 0 \le x \le b = s(0), \\
(iii) & u(0,t) = g(t), & 0 < t \le T, \\
(iv) & u(s(t),t) = 0, & 0 < t \le T, \\
(v) & {}_0^C D^\alpha s(t) = -u_x(s(t),t), & 0 < t \le T,
\end{array} \tag{7}$$

and the second one by using the Riemann-Liouville derivative:

$$\begin{array}{lll}
(i) & \frac{\partial}{\partial t} u(x,t) = \lambda \frac{\partial}{\partial x}\left({}_0^{RL} D_t^{1-\alpha} \frac{\partial}{\partial x} u(x,t)\right), & 0 < x < s(t),\ 0 < t < T, \\
(ii) & u(x,0) = f(x), & 0 \le x \le b = s(0), \\
(iii) & u(0,t) = g(t), & 0 < t \le T, \\
(iv) & u(s(t),t) = 0, & 0 < t \le T, \\
(v) & \frac{d}{dt} s(t) = -{}_0^{RL} D_t^{1-\alpha} \frac{\partial}{\partial x} u(x,t)\Big|_{(s(t),t)}, & 0 < t \le T.
\end{array} \tag{8}$$

In the limit case ($\alpha = 1$) both problems coincide with the same classical Stefan problem. Fractional diffusion equations like $(7-i)$ or $(8-i)$) are linked to the modelling of diffusive processes in heterogeneous media. The Riemann–Liouville fractional derivative ${}_0^{RL} D_t^{1-\alpha}$ is the left inverse operator of the fractional Riemann–Liouville integral ${}_0 I_t^{1-\alpha}$, so we can apply ${}_0^{RL} D_t^{1-\alpha}$ to both sides of equation $(7-i)$ obtaining the fractional diffusion equation $(8-i)$. But, if we apply ${}_0^{RL} D_t^{1-\alpha}$ to both sides of the Stefan condition $(7-v)$ we get

$$\frac{d}{dt} s(t) = -{}_0^{RL} D_t^{1-\alpha} \frac{\partial}{\partial x} u(s(t),t)$$

which is not exactly condition $(8-v)$, unless $\alpha = 1$. In fact, the right side of $(8-v)$ is

$$-\,{}_0^{RL} D_t^{1-\alpha} \frac{\partial}{\partial x} u(x,t)\Big|_{(s(t),t)} = -\lim_{x \to s(t)} \frac{\partial}{\partial t} \frac{1}{\Gamma(\alpha)} \int_0^t (t-\tau)^{\alpha-1} \frac{\partial}{\partial x} u(x,\tau) d\tau. \tag{9}$$

The aim of this paper is to show explicit solutions (in fact, similarity solutions which are given in terms of Wright functions) to problems (7) and (8) respectively, and prove that they are different, which clearly implies that the "fractional Stefan conditions" $(8-v)$ and $(7-v)$ are different and that for fractional derivatives some limits like (9) are not commutative.

The concentration compactness principle for p–Fractional Laplacian in unbounded domains and its applications.

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Abstract

The famous concentration-compactness principle (CCP) due to Lions [1] in 80's is the key to solve problems with lack of compactness in Sobolev embbedings. This principle was originally formulated for critical problems in bounded domain and later extended to deal with critical problem in unbounded domains by Chabrowski [2], is of prime importance since it describes the concentration by a weighted sum of Dirac masses and the loss of mass by measures "supported at infinity". In this talk we show an extension of the refined concentration-compactness at infinity for a nonlocal operator (Fractional Laplacian) and its applications to prove existence of solutions for critical equations in unbounded domains. Joint work Julián Fernández Bonder (UBA-IMAS) and Nicolas Saintier (UBA-IMAS).

Reference

1. [1] P.-L. Lions. *The concentration-compactness principle in the calculus of variations. The limit case. I. Rev. Mat. Iberoamericana*, 1(1):145-201, 1985.
2. [2] J. Chabrowski. *Concentration-compactness principle at infinity and semilinear elliptic equations involving critical and subcritical Sobolev exponents. Calc. Var. Partial Differential Equations*, 3(4):493-512, 1995.

Regularity results for the near field refractor problem

F. Tournier

Abstract

We consider the near field refractor problem with bounded densities and natural convexity assumptions on the domain Ω and the target set $\Omega^\star$ which are subsets of S^{n-1}. We prove C^1 regularity of generalized solutions in the sense of Alexandrov. We consider indexes of refraction κ both bigger and smaller than 1.

The two membranes problem for fully nonlinear operators

Hernán Vivas

Abstract

Motivated by a model from mathematical finance, we consider a version of the two membranes problem for two different fully nonlinear operators. For the case of two different operators of the same order, the only result available is the Hölder regularity obtained by Caffarelli, De Silva and Savin. We prove $C^{1,\alpha}$ regularity of the solution pair for (concave or convex) operators satisfying a sort of compatibility condition and $C^{1,1}$ regularity for the case of the Pucci extremal opeartors, which is optimal.

The precise statement of the problem we consider is the following: given two functions $u_0, v_0 \in C^\gamma(\partial B_1)$ and $f, g \in C^\gamma(B_1)$ for some $\gamma \in (0,1)$, we want to study the solutions u and v of

$$\begin{cases} u \geq v & \text{in } B_1 \\ F(D^2u) \leq f(x) & \text{in } B_1 \\ G(D^2v) \geq g(x) & \text{in } B_1 \\ F(D^2u) = f(x) & \text{in } B_1 \cap \Omega \\ G(D^2v) = g(x) & \text{in } B_1 \cap \Omega \end{cases} \tag{10}$$

where

$$\Omega := \{u > v\},$$

F is convex and

$$G(X) = -F(-X).$$

Equation (10) models a so called "bid and ask" situation in which we have an asset, a seller (represented by u) and a buyer (represented by v). The price of the asset is random and the transaction will only take place when u and v "agree on a price", i.e. when $u = v$. Moreover, we want to model the expected earnings of u and v, assuming that their strategy is optimal.

One can think of this problem as having two different (although related) features: on one hand, we have an "obstacle type" situation, in which u tries to maximize gain with v being an obstacle and vice versa (v minimizing cost and u being an obstacle), hence the constraint $u \geq v$. But perhaps more interesting is the special relation between u and v. Because of the "bid and ask" nature of the model, the Bellman type equations that govern the behavior of our solutions are closely related and it is precisely this feature which opens a way to get regularity even though the operators are different.

Existence and Uniqueness for Parabolic Problems with Caputo Time Derivative

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Abstract

In this talk we are interested in evolution equations whose general form is

$$\partial_t^\alpha u + F(x,t,u,Du,D^2u,\updownarrow(u)) = 0 \quad \text{in } Q, \tag{11}$$

where $Q := \mathbb{R}^N \times (0,+\infty)$, complemented with the initial condition

$$u = u_0 \quad \text{in } \mathbb{R}^N \times \{0\}, \tag{12}$$

with $u_0 \in C(\mathbb{R}^N)$ bounded. In (11), the function $F \in C(\mathbb{R}^N \times \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}^N \times \mathbb{R}^{N\times N} \times \mathbb{R})$ is an elliptic nonlinearity. It is also considered as integro-differential operator in the space variable which, at the introductory level can be considered through the model form

$$\mathcal{I}(u,x) = \int_{\mathbb{R}^N} [u(x+z) - u(x) - \mathbf{1}_B \langle Du(x), z\rangle]\nu(dz),$$

where ν is a nonnegative measure whose basic assumption is the *Lévy integrability condition*

$$\int_{\mathbb{R}^N} \min\{1, |z|^2\}\nu(dz) < +\infty.$$

The main particularity of (11) is the presence of an integro-differential operator in the time variable instead of the classical first-order time derivative arising in the standard parabolic approach. More specifficaly, for $\alpha \in (0,1)$ fixed, D_t^α denotes the Caputo fractional time derivative of α-th order defined as

$$\partial_t^\alpha u(t) = \Gamma(1-\alpha)^{-1} \int_0^t (t-\xi)^{-\alpha} u'(\xi) d\xi,$$

where Γ denotes the Gamma function.

We are interested in the well-posedness of fully nonlinear Cauchy problem (11)-(12) in which the time derivative is of Caputo type. We address this question in the framework of vicosity solutions, obtaining the existence via Perron's method, and comparison for bounded sub and supersolutions by a suitable regularization through inf and sup convolution in time. As an application, we prove the steady-state large time behavior in the case of proper nonlinearities and provide a rate of convergence by using the Mittag-Leffler operator.

Poster Session

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Abalos, Julio	A necessary condition for strong hyperbolicity of general first order systems
Acinas, Sonia	Periodic solutions of Euler-Lagrange equations in an anisotropic Orlicz-Sobolev space setting
Alliera, Carlos	Control de la ecuacin diferencial de Gompertz
Averbuj, Corina	Endogenous growth economic model with delay in the control variable. Dynamic programming approach
Balderrama, Roco	On fixed point theorems in partially ordered Banach spaces and applications
Ben, Roberto	Breathers and Variational Approximations for a Nonlocal NLS Equation
Blanc, Pablo	A lower bound for the principal eigenvalue of fully nonlinear elliptic operators
Bollati, Julieta	Mathematical model for acid water neutralization with anomalous and fast diffusion
Cantizano, Natal	Multiplicidad de Soluciones para el Laplaciano Fraccionario
Casari-Rampasso, Giane	A priori estimates for viscosity solutions of fully nonlinear parabolic equations
Dboli, Alberto	Necessary and sufficient conditions for the existence of positive $(c,\omega)-$ periodic solutions of a Nicholson type delay system
Espindola, Mara	The Linearization of the Schrdinger Equation
Espinoza, Juan	Modelado numrico de Tsunamis en Venezuela. Caso: Macuto
Etcheverry, Javier	Nonlinear PDEs in applied electromagnetism
Goos, Demian	On the nonlinear nature of homogeneous PDEs of fractional order
Itovich, Griselda	Global stability analysis of a second order delay dierential equation
Rey, Carolina	Ecuacin elptica con exponente crtico en una regin de S^3 invariante por la accin de T^2
Ritorto, Antonella	A Nonlocal Optimal Partition Problem

Author	Title
Rodriguez Carranza, Alexis	Simultaneous exact controllability of wave sound propagations in multilayer media
Rodriguez Cartabia, Mauro	A Game Theory Model For Wealth Distribution
Rodriguez, Andrei	Loss of boundary conditions for two classes of nonlinear parabolic equations with dominating gradient terms: (second-order) fully nonlinear and fractional cases
Rubio, Marcelo	The initial value problem of Force-free Electrodynamics in Euler Potentials
Scarola, Cristian	Inverse Problem For Energy Dependent Potentials
Varas Perez, Elder	Energy cascade in turbulent flows using the sabra shell mode