Abstracts of Posters

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A necessary condition for strong hyperbolicity of general first order systems.

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We study strong hyperbolicity of first order partial differential equations for systems with differential constraints. In those cases, the number of equations is larger than the unknown fields, therefore, the standard hyperbolicity does not directly apply. To deal with this problem one introduces a new tensor, called a hyperbolizer, which selects a subset of equations so that they might be used as evolution equations for the unknowns. Different hyperbolizers, may or may not lead to strongly hyperbolic evolution systems.

To sort-out this issue, we look for a condition which is independent of the hyperbolizers but which, if satisfied, implies the non-existence of strongly hyperbolic reductions [1]. We look at the singular value decomposition of the whole system and show that if the principal symbol is appropriately perturbed (via a parameter ε), we obtain perturbed characteristic surfaces with given perturbed singular values. If any of the singular values is order $O\left(\varepsilon^l\right)$, with $l \geq 2$, hyperbolicity breaks down. In addition, we notice that the calculations at orders $O\left(\varepsilon^0\right)$ and $O\left(\varepsilon^1\right)$ can be done in a covariant way.

These results are reached by connecting the perturbation of singular values with the diagonalization of square matrices, leading to a different and simpler proof of Kreiss's matrix theorem. We then extend these conclusions to systems with constraints.

Finally we apply the results to examples in physics, such as Force-Free Electrodynamics in Euler potentials form and charged fluids with finite conductivity. We find that they are only weakly hyperbolic.

References

[1] Abalos, Fernando. "A necessary condition for strong hyperbolicity of general first order systems." arXiv preprint arXiv:1707.05011 (2017).

Periodic solutions of Euler-Lagrange equations in an anisotropic Orlicz-Sobolev space setting

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Let $\Phi : \mathbb{R}^d \to [0, +\infty)$ be a differentiable, convex function such that $\Phi(0) = 0$, $\Phi(y) > 0$ if $y \neq 0$, $\Phi(-y) = \Phi(y)$, and $\lim_{|y|\to\infty} \frac{\Phi(y)}{|y|} = +\infty$.

For $T > 0$, we assume that $F : [0, T] \times \mathbb{R}^d \to \mathbb{R}^d$ with F and $\nabla_x F$ Carathéodory functions satisfying

$$|F(t, x)| + |\nabla_x F(t, x)| \leq a(x)b(t), \text{ for a.e. } t \in [0, T],$$

where $a \in C\left(\mathbb{R}^d, [0, +\infty)\right)$ and $0 \leq b \in L^1([0, T], \mathbb{R})$.

Our goal is to set conditions that guarantee existence of solutions of the problem

$$\begin{cases} \frac{d}{dt}\nabla\Phi(u'(t)) = \nabla_x F(t, u(t)), & \text{for a.e. } t \in (0, T), \\ u(0) - u(T) = u'(0) - u'(T) = 0, \end{cases} \tag{P_Φ}$$

minimizing the functional

$$I(u) := \int_0^T \Phi(u'(t)) + F(t, u(t))\ dt,$$

on the set $H := \{u \in W^1L^\Phi | u(0) = u(T)\}$, where W^1L^Φ is the anisotropic Orlicz-Sobolev space associated to Φ.

We denote by Φ^* the complementary function of Φ in the sense of convex analysis.

We say that $\Phi \in \Delta_2$, if there exists a constant $C > 0$ such that

$$\Phi(2x) \leq C\Phi(x) + 1, \quad x \in \mathbb{R}^d.$$

We write $\Phi_0 \prec \Phi$ if for every $k > 0$ there exists $C = C(k) > 0$ such that

$$\Phi_1(x) \leq C + \Phi_2(kx), \quad x \in \mathbb{R}^d.$$

The following is our main theorem which contains, as a particular case, known results of existence of periodic solutions of p-laplacian and (p, q)-laplacian systems.

Theorem 1. *Let $\Phi^\star \in \Delta_2$.*

1. *If there exist Φ_0 with $\Phi_0 \prec \Phi$ and a function $d \in L^1([0, T], \mathbb{R})$, with $d \geq 1$, such that*

$$\Phi^\star(d^{-1}(t)\nabla_x F) \leq \Phi_0(x) + 1 \quad \textit{and} \quad \lim_{|x|\to\infty} \frac{\int_0^T F(t, x)\ dt}{\Phi_0(2x)} = +\infty,$$

then I attains a minimum on H.

2. *If u is a minimum and $d\left(u', L^\infty([0, T], \mathbb{R}^d)\right) < 1$, then u is solution of (P_Φ).*

We also discuss conditions under which $d(u', L^\infty([0, T], \mathbb{R}^d)) < 1$ is satisfied.

Now, we are exploring the possibility of addressing Hamiltonian systems by the dual method, instead of the direct one applied to Euler-Lagrange equations, in an anisotropic Orlicz-Sobolev framework.

Control de la ecuación diferencial de Gompertz

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La siguiente ecuación con retardo, conocida como Ecuación de Gompertz [1], representa el crecimiento tumoral:

$$N'(t) = rN(t) \ln\left(\frac{K}{N(t-\tau)}\right) \tag{1}$$

De condición inicial $N(t) = \varphi(t), \quad t \in [-\tau, 0]$ para cierta $\varphi \in C([-\tau, 0], \mathbb{R}^+)$.
El equilibrio estable del sistema (1) es: $N_0^* = K$ donde $K > 0$ es la capacidad de carga y representa el tamaño máximo que puede alcanzar el tumor.
Para disminuir el valor de ese equilibrio se proponen modelos de control (en la práctica representan un tratamiento contra el tumor) como el siguiente:

$$\begin{cases} N'(t) = rN(t)\left\{\ln\left(\frac{K}{N(t-\tau)}\right) - cu(t)\right\} \\ u'(t) = -au(t) + b\ln(N(t)) \\ N(t) = \varphi(t), \ t \in [-\tau, 0], \\ u(0) = u_0 \end{cases} \tag{2}$$

en el cual los parámetros adicionales $a, \ b, \ c > 0$ y la función $\varphi \in C^1([-\tau, 0])$ es estrictamente positiva, y $u_0 > 0$.
El equilibrio del sistema con control ahora es:

$$\boxed{N_1^* = K^{\frac{a}{a+cb}} < K, \ u^* = \frac{b\log(K)}{a+cb}}$$

Buscaremos condiciones para determinar la estabilidad de este nuevo equilibrio mediante funcionales de Lyapunov.
Además, analizaremos numéricamente ejemplos de tumores como los que se muestran en [2] donde se evidencia el efecto de este control.

Referencias

[1] Gompertz, Benjamin, *On the Nature of the Function Expressive of the Law of Human Mortality, and on a New Mode of Determining the Value of Life Contingencies*,(1825).

[2] Laird, Anna Kane.*Dynamics of Tumor Growth*, British Journal of Cancer. 18 (3): pp 490–502 (1964).

Endogenous growth economic model with delay in the control variable.
Dynamic programming approach

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In this working paper, a generalized neoclassic model is studying, where the consumption is endogenous and a temporal delay between the production and the real investment is proposed.

1 Introduction

From the work of Solow (1956), where the economic growth model assumed a constant saving rate and consumption was exogenous, economic theory has been developed in order to understand the macroeconomic temporal dynamics; one way of approaching this study is to propose, as in general equilibrium theory, ordered preferences (utility functions) on which savings and investment decisions are made.

Thus, more general endogenous growth models emerged in the neoclassical theory, where the problem consisted in finding a consumption trajectory that maximized some utility, subject to constraints given by a dynamic system that shows the temporal evolution of the distribution of production between consumption and investment (Fleming, Rishel, 1975).

Understanding that the dynamics in investment is central to the study of aggregate fluctuations, our fundamental interest will be to study optimal control models, where the investment is defined as an operator $\Upsilon_{(t,\mathbf{X}(t))}(\cdot)$, dependent on all the factors that make the construction of capital over time and that also "disaggregates" productive capital according to its maturity time. Thus the functions will not only show the capacity of the economy to make investments effective, but also their productive diversity.

1.1 The problem

This working paper has been developed to study a class of state contrained optimal control problem with distributed delay in the control variable and the associated Hamilton-Jacobi- Bellman equation, extending (Averbuj-Martinez, 2014).Then, our main objetive will be maximize a functional of the form $J(c) = \int\limits_0^{+\infty} u(c)e^{-\rho t}dt$, with the optimal control of one dimensional delay differential equation as:

$$f[k(t)] - c(t) = \int_t^{t+T} \Upsilon_{(t,\mathbf{X}(t))}(\tau)d\tau$$

Title: On fixed point theorems in partially ordered Banach spaces and applications.
Author: Rocío Balderrama

In this work we establish some fixed point theorems of nonlinear mixed monotone operators in Banach spaces partially ordered by a cone. As its application, some easy-to-verify sufficient conditions for the existence of nonlinear differential equations are obtained. In addition, we also provide some applications to biological models to illustrate our main results.

Breathers and Variational Approximations for a Nonlocal NLS Equation

Poster author: Roberto I. Ben (Universidad Nacional de General Sarmiento; rben@ungs.edu.ar)

The coupled system (1) models the interaction between the complex amplitude u of the electric field of a polarized laser beam propagating through a nematic liquid crystal sample, and the director field angle θ describing the macroscopic orientation of the molecules of the liquid crystal.

$$i\partial_z u + \frac{1}{2}\nabla_{xy}^2 u + sin(2\theta)u = 0, \tag{1a}$$

$$\nu\nabla_{xy}^2\theta - qsin(2\theta) = -2\left|u\right|^2 cos(2\theta), \tag{1b}$$

ν and q are real constants and the variable z represents the optical axis of light propagation, playing here the role of time, as $u(x, y, z)$ and $\theta(x, y, z)$ are given at $z = 0$. Well posedness of this problem and existence of stationary solutions are studied in [BPSR17].

A discrete version, which models laser beam propagation in a thin film planar waveguide of nematic liquid crystals, subject to a periodic transverse modulation along y and across x, was proposed by Fratalocchi and Assanto [FA05]. The model is a discrete nonlinear Scrödinger equation (DNLS) with a cubic Hartree nonlocality:

$$\dot{u}_n = \delta i\left(u_{n+1} + u_{n-1} - 2u_n\right) + 2\gamma \tanh\frac{\kappa}{2} i \sum_{m\in\mathbb{Z}} e^{-\kappa|m-n|}\left|u_m\right|^2 u_n, n \in \mathbb{Z},\ t > 0. \tag{2}$$

where δ and γ are real numbers and $\kappa > 0$. Breathers solutions (discrete solitons) to this DNLS are studied in [BBP17] and [BCMP15] in the anticontinuous case ($\delta = 0$), showing the existence of continuations for δ small enough.

In this poster presentation we analyse, based on a variational approach, some types of analytical approximations to this equation; and we compare them with the respective solutions found numerically in [BCMP15].

A work in progress is also presented, in which we are looking for breathers solutions to a DNLS with cubic and seven grade Hartree nonlinearities. These nonlinearities are a consequence of the third order Taylor expansion applied to sine and cosine in system (1). Relating this study with the results found in [CTCM06] for the cubic-quintic DNLS we can conjecture, under certain conditions, the presence of bifurcations in the amplitude of breathers with equal decay.

Referencias

[BCMP15] R.I. Ben, L. Cisneros Ake, A.A. Minzoni, P. Panayotaros, *Localized solutions for a nonlocal discrete NLS equation*, Phys. Lett. A 379, 17051714 (2015).

[BBP17] R.I. Ben, J.P. Borgna, P. Panayotaros, *Properties of some breather solutions of a nonlocal discrete NLS equation*, Communications in Mathematical Sciences (forthcoming paper).

[BPSR17] J. P. Borgna, P. Panayotaros, C. Sánches de la Vega, D. Rial, *Optical solitons in nematic liquid crystals: model with saturation effects*, preprint.

[CTCM06] R. Carretero-González, J.D. Talley, C. Chong, B.A. Malomed, *Multistable solitons in the cubic-quintic discrete nonlinear Schrödinger equation*, Physica D 216 (2006), pp. 77-89.

[FA05] A. Fratalocchi, G. Assanto, *Discrete light localization in one-dimensional nonlinear lattices with arbitrary nonlocality*, Phys. Rev. E 72, 066608 (2005)

A LOWER BOUND FOR THE PRINCIPAL EIGENVALUE OF FULLY NONLINEAR ELLIPTIC OPERATORS

PABLO BLANC

In this poster we present a new technique to obtain a lower bound for the principal Dirichlet eigenvalue of a fully nonlinear elliptic operator. We illustrate the construction of an appropriate radial function required to obtain the bound in several examples. In particular we use our results to prove that

$$\lim_{p\to\infty} \lambda_{1,p} = \lambda_{1,\infty} = \left(\frac{\pi}{2R}\right)^2$$

where $\lambda_{1,p}$ and $\lambda_{1,\infty}$ are the principal eigenvalue for the homogeneous p-laplacian and the homogeneous infinity laplacian respectively.

The article in which the poster is based is available at arxiv.org/pdf/1709.02455.pdf

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Mathematical model for acid water neutralization with anomalous and fast diffusion

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Abstract

In this contributed talk we model the neutralization of an acid solution in which the hydrogen ions are transported according to Cattaneo's diffusion. The latter is a modification of classical Fickian diffusion in which the flux adjusts to the gradient with a positive relaxation time. Accordingly the evolution of the ions concentration is governed by the hyperbolic telegraph equation instead of the classical heat equation. We focus on the specific case of a marble slab reacting with a sulphuric acid solution and we consider a one-dimensional geometry. We show that the problem is multi-scale in time, with a reaction time scale that is larger than the diffusive time scale, so that the governing equation is reduced to the one-dimensional wave equation. The mathematical problem turns out to be a hyperbolic free boundary problem where the consumption of the slab is described by a nonlinear differential equation. Global well posedness is proved and some numerical simulations are provided.

Keywords: neutralization, reaction kinetics, multi-scale modeling, free boundary problem, anomalous diffusion

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Multiplicidad de Soluciones para el Laplaciano Fraccionario

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En este trabajo mostramos la existencia de al menos tres soluciones no triviales, de la ecuación elíptica $(-\Delta)^s u = |u|^{2_s^*-2}u + \lambda f(x,u)$ en un dominio de borde suave Ω en $\mathbb{R}^N$ cuyo borde $\partial\Omega$ cumple con las condiciones homogéneas de Dirichlet. Donde $2_s^* = \frac{n2}{n-s2}$ es el exponente crítico fraccionario de Sobolev y $(-\Delta)^s u := C(n,s)\, p.v. \int_\Omega \frac{u(x)-u(y)}{|x-y|^{n+2s}} dy$ es el Laplaciano Fraccionario, donde $C(n,s)$ es una constante dimensional que depende de n y s. La prueba está basada en argumentos variacionales y el método clásico de concentración por compacidad.

A priori estimates for viscosity solutions of fully nonlinear parabolic equations

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Abstract

Since the foundation of the modern analysis of partial differential equations, the regularity theory for solutions has been one of the main subjects of research. In particular, a substantial class of equations involve operators whose ellipticity degenerates along an *a priori* unknown region: the free boundary. Since this region can also vary according to the solution itself, regularity become a delicate issue depending on the diffusion control near such a region.

In special, we are interested in the study of fully nonlinear parabolic equation with strong absorption and gradient dependence, as follows

$$\begin{cases} \partial_t u - F(D^2(u)) = -\gamma u^{\gamma-1}\Gamma(|\nabla u|)\chi_{\{u>0\}}, & \text{in} \quad Q_1 = B_1(0)\times(-1,0]\subset\mathbb{R}^{d+1} \\ u(x,t) = 1, \quad \text{on} \quad \partial B_1(0)\times(-1,0], \\ u(x,-1) = u_0(x) \quad \text{in} \quad B_1(0) \end{cases} \tag{PDE}$$

where $F\colon \mathcal{S}(d)\to\mathbb{R}$ is (λ,Λ)-elliptic, and convex, $\mathcal{S}(d)$ is the set of all symmetric $d\times d$ real matrix, $1<\gamma<2$, and $\Gamma\colon[0,\infty)\to\mathbb{R}$ is defined by $\Gamma(t)=1+t^m$, $0<m<2-\gamma$.

The purpose of this work is to obtain *a priori* estimates for viscosity solutions, that are, in some sense, uniform with respect to some analytical properties of the right-hand side of (PDE). Hence, we generalize some recent results of Montenegro-Queiroz-Teixeira and Choe-Weiss.

This is part of a work joint Olivaine S. de Queiroz.

Necessary and sufficient conditions for the existence of positive (c,ω)-periodic solutions of a Nicholson type delay system

P. Amster, A. Déboli and M. Pinto

In this work we present, by means of topological degree methods, necessary and sufficient conditions for the existence of at least one positive (c,ω)-periodic solution of a Nicholson type delay system

$$\begin{cases} x_1'(t) = -\delta_1(t)x_1(t) + \beta_1(t)x_2(t) + p_1(t)x_1(t-\tau_1)e^{-a_1(t)x_1(t-\tau_1)} \\ x_2'(t) = -\delta_2 t)x_2(t) + \beta_2(t)x_1(t) + p_2(t)x_2(t-\tau_2)e^{-a_2(t)x_2(t-\tau_2)} \end{cases} \tag{1}$$

where $\delta_i, \beta_i, p_i \in C(\mathbb{R}, \mathbb{R}^+)$ are ω-periodic, $a_i \in C(\mathbb{R}, \mathbb{R}^+)$ are $(1/c, w)$-periodic for $i = 1, 2$ and τ_i are positive constants.

The class of (c,ω) functions includes periodic, anti-periodic, Bloch functions but also unbounded functions when $|c| \neq 1$. More specifically, for a given pair (c,ω) such that $c \in \mathbb{C}\backslash\{0\}, \omega > 0$, we shall say that $g \in C(\mathbb{R}, \mathbb{C})$ is a (c,ω)-periodic function if $g(t+\omega) = cg(t)$ for all $t \in \mathbb{R}$ (see e.g. [2], [3]).

Systems of type (1) were used, for example, in marine protected areas and to describe the dynamics of the B-cells of the lymphocytic leukemia ([1],[4]).

References

[1] L. Berezansky, L. Idels, L. Troib. Global dynamics of Nicholson-type delay system with applications. Nonlinear Anal. Real World Appl. 12 (2011), 436–445.

[2] M. Pinto. Ergodicity and Oscillations. Conference in Universidad Católica del Norte. 2014.

[3] M. Pinto. Pseudo-almost periodic solutions of neutral integral and differential equations with applications. Nonlinear Anal. 72(12)(2010), 4377–4383.

[4] H. Smith. An Introduction to Delay Differential Equations with Applications to the Life Sciences, Springer-Verlag, New York, 2011.

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Modelado numérico de Tsunamis en Venezuela. Caso: Macuto

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Se presenta una simulación numérica para el tsunami ocurrido en Miranda-Venezuela el 29 de octubre de 1900, un fenómeno de magnitud local 7.7 que afectó a zonas como Macuto (Venezuela) y Trinidad y Tobago. Este trabajo discute la dinámica de la onda producida por un sismo con base en las ecuaciones que describen la mecánica de fluidos (Navier-Stokes) se plantean las ecuaciones diferenciales que modelan la propagación del tsunami. El objeto de este estudio es verificar las salidas del modelo con datos de Alborán-España y generar una GUI para que el modelado sea más amigable.

Este trabajo es una ampliación del presentado en las Jornadas de Investigación y Extension 2014, Facultad de Ciencias de la Universidad Central de Venezuela, Caracas-Venezuela (MyT-4-335).

Palabras claves: Tsunami, batimetría, línea de costa, fluidos

Nonlinear PDEs in applied electromagnetism

Javier Etcheverry, Tenaris R&D

Applied electromagnetism is an inexhaustible source of linear and nonlinear PDE problems. We will focus here on problems associated with the non-destructive evaluation of ferromagnetic materials. Typical applications are relatively low frequency (which means that the full hyperbolic Maxwell equations system can be simplified) and are usually well described by the so-called induced currents approximation (parabolic/elliptic system). To be specific, we can consider the equation

$$\sigma \frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \nabla \times \boldsymbol{H}$$

where σ is the electrical conductivity (positive on conductors, zero in insulators, air, etc.), and vectors ***B***, ***H*** are the divergence free magnetic induction and the magnetic field, respectively.

For non-ferromagnetic materials $\boldsymbol{B}=\mu_0 \boldsymbol{H}$, with μ_0 a positive constant (the vacuum magnetic permeability), and the equation above is linear. Instead, for ferromagnetic materials the relationship between ***B*** and ***H*** is extremely complex, which gives rise to many different approximation ideas. For instance, it can be modelled as a vector multibranch nonlinearity (where the actual $\boldsymbol{B}(\boldsymbol{H})$ value depends both on past history and on the sign of the variation), as a nonlinear discontinuous function (thus giving rise to free boundary problems), as a nonlinear smooth function $\boldsymbol{B}=\mu(||\boldsymbol{H}||)\,\boldsymbol{H}$ (where $\mu(||\boldsymbol{H}||)$ is the magnetic permeability of the material, usually much larger than μ_0), etc.

Even in this last very simplified form, permeability can be given many different parameterizations, like

$$\mu - \mu_0 = \frac{1}{a\|\boldsymbol{H}\| + b} .$$

In this poster we present several nonlinear elliptic/parabolic problems of interest in applications, which may lead to free boundary problems according to modeling choices, with the goal of fostering interest on applied problems, and help illuminating from the theory some of the very complex issues involved in the techniques.

On the nonlinear nature of homogeneous PDEs of fractional order

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A linear fractional Schrödinger equation is studied, in which the differentiation operators are of fractional Caputo type:

$$i\hbar\frac{\partial^\beta\Phi}{\partial t^\beta}(x,t) = -\frac{\hbar^2}{2\mu}\frac{\partial^{2\alpha}\Phi}{\partial x^{2\alpha}}(x,t) + V(x)\Phi(x,t) \qquad \forall x\in\mathbb{R}, t\in\mathbb{R}^+$$

where μ is the mass of the particle, V is the potential, $\hbar$ is Planck's reduced constant and $\alpha,\beta\in\left(0,\frac{1}{2}\right]$ are constants.

The solution to this equation within the potential well (both, the finite as well as the infinite one) is derived in order to visualize the nonlinear nature of the equation.

A numerical scheme is used in order to solve the arising nonlinear differential equation and different properties of the general model and its solutions are presented.

References

[1] K. Diethelm, *The Analysis of Fractional Differential Equations*, Springer, Heidelberg (2004).

[2] J. W. Hanneken, D. M. Vaught, B. N. Narahari Achar, *Enumeration of the Real Zeros of the Mittag–Leffler Function* $E_\alpha(z), 1<\alpha<2$, Advances in Fractional Calculus, pp 15–26, (2007).

[3] H.J. Haubold, A.M. Mathai, R.K. Saxena, *Mittag-Leffler Functions and Their Applications*, *J. Appl. Math.*, 298–628, (2011).

[4] E. Rebhan, *Theoretische Physik: Quantenmechanik*, Spektrum Akademischer Verlag, (2008).

[5] V. I. Kukulin, V. M. Krasnopolsky,J. Horacek, *Theory of Resonances – Principles and Applications*, Kluwer Academic Publishers, (1988).

Global stability analysis of a second order delay differential equation

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It is considered a second order differential equation with one delay

$$\ddot{x} + \gamma x = f(x(t-\tau)), \tag{1}$$

where $f(x) = \alpha x + \beta x^2$, $\gamma, \tau > 0$ and $\alpha, \beta \neq 0$. This model was analyzed in the work of Campbell and LeBlanc (1998) for $\gamma = 2.5$ and $\beta = 0.9$ in relation with a 1:2 resonance. Now, the original model is modified, just to include four parameters, counting the delay τ. Then, equation (1) can be written as

$$\dot{y}_1 = -\gamma y_2 + f(y_2(t-\tau)), \quad \dot{y}_2 = y_1, \tag{2}$$

and the equilibrium points result $Y_1 = (0,0)$ and $Y_2 = \left(0, (\gamma-\alpha)\,\beta^{-1}\right)$. Two approaches were combined to carry out the dynamic study of (2) analytically: one, which is more usual and we call it "in time domain" (Bellman and Cooke, 1963), for simplicity, and other which uses concepts of control theory, named as "frequency domain" (Moiola and Chen, 1996). Both points of view were applied, exploding the advantages of each one. Besides, the Dde-Biftool software (Engelborghs et *al.*, 2002) was employed to check results and show other complex stability issues.

- **Equilibrium points stability**

To analyze the stability of the equilibrium points is necessary to find the zeroes of an equation, which in the case of the trivial one gives $\lambda^2 + \gamma - \alpha e^{-\lambda\tau} = 0$. This follows from the linearization of (2), evaluated at Y_1. If all its roots have negative real parts then Y_1 results asymptotically stable. Changing variables: $z = \lambda\tau$, the last equation becomes $P_1(z) = e^z z^2 + e^z \gamma\tau^2 - \alpha\tau^2 = 0$, this means that one has to locate the roots of an exponential polynomial. Thus, the next result can be proved:

Theorem 1 *It is considered the equation $P_1(z) = 0$, where $\gamma, \tau > 0$ and $\alpha \neq 0$. Let $\tau > 0$ and $r = \left(\frac{\pi}{\tau}\right)^2$. It is supposed that $-\gamma < \alpha < \gamma$, $\gamma \neq k^2 r$, $k \in \mathbb{N} \cup \{0\}$. As $\gamma \in \bigcup_{k=0}^{\infty} (k^2 r, (k+1)^2 r)$, it follows that*
(a) If $k^2 r < \gamma < (k+1)^2 r$ and k is even, then P_1 has all its roots with negative real parts if

$$\gamma - (k+2)^2 r < \alpha < \gamma - k^2 r, \quad 0 < \alpha < -\gamma + (k+1)^2\, r.$$

(b) If $k^2 r < \gamma < (k+1)^2 r$ and k is odd, then P_1 has all its roots with negative real parts if

$$\gamma - (k+1)^2 r < \alpha < \gamma - (k-1)^2 r, \quad -\gamma + k^2 r < \alpha < 0.$$

Corollary 1 *The trivial equilibrium of (1) is asymptotically stable if its parameters satisfy the conditions given in Theorem 1.*

- **Hopf curves and other singularities related with the trivial equilibrium Y_1**

1. Fixing a value of the parameter τ (in the $\gamma - \alpha$ plane)

To detect the Hopf bifurcations related with Y_1, one must solve the system $-\omega^2 + \gamma - \alpha\cos\omega\tau = 0$, $\alpha \sin\omega\tau = 0$. Therefore, one obtains the Hopf bifurcation curves, which in this case are the straight lines

$$\bar{\alpha}_k(\gamma) = (-1)^k\gamma + (-1)^{k+1}k^2 r, \text{ where } \quad k \in \mathbb{N} \text{ and } r = \left(\frac{\pi}{\tau}\right)^2. \tag{3}$$

Moreover, in the $\gamma - \alpha$ plane infinite points can be found where resonant double Hopf points appear, which result as the intersection of two $\bar{\alpha}_k$ lines, choosing an odd k_1 and an even $k_2 \neq 0$.

2. Fixing a value of the parameter γ (in the $\tau - \alpha$ plane)

Ecuación elíptica con exponente crítico en una región de $\mathbb{S}^3$ invariante por la acción de $\mathbb{T}^2$

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Abstract

Dada la siguiente ecuación elíptica con exponente crítico

$$\Delta_{\mathbb{S}^3} U = -(U^5 + \lambda U)$$

estudiaremos la multiplicidad de soluciones positivas en una región Ω de $\mathbb{S}^3$ invariante por la acción natural del toro $\mathbb{T}^2$ que se anulan en la frontera $\partial\Omega$. Si suponemos que la función $U : \Omega \to \mathbb{R}$ es también invariante por la acción de $\mathbb{T}^2$, entonces $U = u(\theta)$ para alguna función $u : [0, \theta_1] \to \mathbb{R}$ que es solución de la siguiente ODE:

$$\left\{ \begin{array}{rcl} u''(\theta) + 2\frac{\cos(2\theta)}{\sin(2\theta)} u'(\theta) & = & \lambda \left(u(\theta)^5 - u(\theta) \right), \quad u > 0 \ \text{ en } (0, \theta_1) \\ u'(0) & = & 0 \\ u(\theta_1) & = & 0 \end{array} \right. \tag{1}$$

A partir del estudio de este nuevo problema vamos a demostrar que el número de soluciones de (1) aumenta a medida que λ tiende a $-\infty$, dando una respuesta a un caso particular de un problema abierto propuesto por Brezis y Peletier en [1].

References

[1] H. Brezis and L. A. Peletier *Elliptic equations with critical exponent on spherical caps of* $\mathbb{S}^3$ J. Anal. Math., No. 98, pp. 279-316, 2006.

A NONLOCAL OPTIMAL PARTITION PROBLEM

ANTONELLA RITORTO

Abstract. We prove an existence result for an optimal partition problem of the form

$$\min\{F_s(A_1,\dots,A_m)\colon A_i \in \mathcal{A}_s,\ A_i\cap A_j=\emptyset \text{ for } i\neq j\},$$

where F_s is a cost functional with suitable assumptions of monotonicity and lower semicontinuity, $\mathcal{A}_s$ is the class of admissible domains and the condition $A_i\cap A_j=\emptyset$ is understood in the sense of Gagliardo s-capacity, where $0<s<1$. Examples of this type of problem are related to fractional eigenvalues. We also demonstrate some type of convergence of the s-minimizers to the minimizer of the problem with $s=1$, studied in *Existence results for some optimal partition problems*, Bucur-Buttazzo-Henrot, 1998.

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Simultaneous exact controllability of wave sound propagations in multilayer media

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Abstract

In this work we study one problem of mathematical interest for their applications in several topics in Applied Science. We study simultaneous controllability of a pair of systems which model the evolution of sound in a compressible flow considered as a transmission problem. We show the well posed of the problem. Furthermore provided appropriate conditions in the geometry of the domain are valid and suitable assumptions on the fluid, is possible to conduce the pair of systems to the equilibrium in a simultaneous way using only one control.

References

[1] B.V. Kapitonov, and G., *Simultaneous exact controllability for a pair of systems of evolution of sound in a compressible fluid, Journal of Mathematics and System Science*, 3 (2013) 655-658.

[2] Alexis Rodriguez Carranza, *Simultaneous Exact Controllability of Wave Sound Propagations in Multilayer Media.* Journal of Mathematics and System Science 6 (2016) 323-329.

A GAME THEORY MODEL FOR WEALTH DISTRIBUTION

MAURO RODRIGUEZ CARTABIA (IMAS-CONICET AND DM-FCEN-UBA)

There are many works showing that the distribution of wealth in a population follows Gamma or Pareto distributions, with a power law decay. In this work we study a model where wealth exchanges depend on a symmetric zero sum game. After each interaction, players mixed strategies are updated looking for a better outcome in future games. We observe that, when the optimal way of play is a mixed strategy, the wealth distribution follows a Gamma distribution, and the parameters depend on both the variance of the optimal strategies and the amount of wealth interchanged. However, when Nash equilibrium is a pure strategy those players which learn it faster accumulate a significant part of the total wealth.

Abstract: Loss of boundary conditions for two classes of nonlinear parabolic equations with dominating gradient terms: (second-order) fully nonlinear and fractional cases.

Andrei Rodríguez

October 11, 2017

We present a contribution to the study of qualitative properties of viscosity solutions of nonlinear parabolic equations whose rate of growth with respect to the gradient variable makes the corresponding term the dominant one in the equation. Specifically, we show that the phenomenon of *loss of boundary conditions* (LOBC, for short; defined below) occurs for two model problems with prescribed boundary and initial value data.

The first is the fully nonlinear case:

$$u_t - \mathcal{M}^-(D^2u) = |Du|^p \quad \text{in } \Omega \times (0,T), \tag{1}$$

$$u|_{\partial\Omega\times(0,T)} = 0, \qquad u(\cdot,0) = u_0 \in C^1(\overline{\Omega}). \tag{2}$$

Here $\Omega \subset \mathbb{R}^N$ is a bounded, smooth domain, $T > 0$; $\mathcal{M}^-$ denotes Pucci's (minimal) extremal operator, $\mathcal{M}^-(X) = \inf\{\mathrm{tr}(AX) | \lambda \mathrm{Id} \leq A \leq \Lambda \mathrm{Id}\}$, where A and X are symmetric $N \times N$ matrices and $0 < \lambda \leq \Lambda$; and $p > 2$. The second model problem is of nonlocal character:

$$u_t + (-\Delta)^s u = |Du|^p \quad \text{in } \Omega \times (0,T), \tag{3}$$

$$u|_{\mathbb{R}^N\setminus\Omega\times(0,T)} = 0, \qquad u(\cdot,0) = u_0 \in C^\beta(\overline{\Omega}), \tag{4}$$

where Ω and T are as before, $(-\Delta)^s$ denotes the well-known fractional Laplacian operator with $s \in (0,1)$, p satisfies

$$s + 1 < p < \frac{s}{1-s}, \tag{5}$$

and $\beta < {}^{p-2s}/_{p-1}$; (5) restricts the value of s to $(0.618\ldots, 1)$, where $0.618\ldots$ is the constant sometimes called *reciprocal golden ratio*. These restrictions, however, are related to our methods and may not be essential.

For each of our model problems, we prove that *a)* there exists a small time $T^* > 0$ depending only on the initial condition u_0 (specifically, on $\|u_0\|_{C^1(\overline{\Omega})}$ and $\|u_0\|_{C^\beta(\Omega)}$, respectively) and universal constants, such that the corresponding viscosity solution satisfies the boundary data in the classical sense (pointwise); and *b)* LOBC occurs depending on a largeness condition for u_0 given in terms of an eigenfunction of $\mathcal{M}^-$ and $(-\Delta)^s$, respectively. Joint work with Alexander Quaas.

Definitions and remarks on our methods. We study equations (1) and (3) from the viewpoint of (continuous) viscosity solutions. In this context we employ the notion of *generalized* or *viscosity* boundary conditions, which amount to stating the equations hold up to the boundary wherever (2) or (4), respectively, fail to hold pointwise. More precisely: for, e.g. (1), a subsolution satisfies the first part of (2) in the viscosity sense if

$$\min\left\{u(x,t), u_t(x,t) - \mathcal{M}^-(D^2u(x,t)) - |Du(x,t)|^p\right\} \leq 0 \quad \text{for all } (x,t) \in \partial\Omega \times (0,T).$$

The corresponding definitions for supersolutions and for sub- and supersolutions of (3) are similar. This notion is motivated by the optimal control problems underlying (1) and (3), and has been used to obtain the existence of solutions defined globally in time, i.e., solving (1) or (3) for any $T > 0$.

The initial value problem of Force-free Electrodynamics in Euler Potentials

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ABSTRACT

It is well known that in the neighborhood of a pulsar or a black hole, the presence of strongly magnetic fields gives rise to the generation of a very diluted plasma. In that region, the electromagnetic field dominates over the matter constituting that plasma, and the resulting uncoupled dynamics is commonly known as Force-free Electrodynamics.
In this talk I will present a recent result [1] about the initial value problem of considering Force-free Electrodynamics in Euler Potentials. We prove that the initial value problem for Force-free Electrodynamics in Euler variables is not well posed, establishing this result by showing that a well-posedness criterion provided by Kreiss [2] fails to hold for this theory, and using a theorem provided by Strang [3].
To show the nature of the problem I will display a particular bounded (in Sobolev norms) sequence of initial data for the Force-free equations such that at any given time as close to zero as one wish, the corresponding evolution sequence is not bounded. Thus, the Force-free evolution is non continuous in that norm with respect to the initial data, implying that this formulation should not be used in numerical simulations or other kinds of approximations for simulating dynamics of accretion disks around spinning black holes. Growing linear perturbations will become arbitrarily stiff as the grid frequency is increased, and furthermore non-linearities can alter that growth making it to become exponential and thus rendering computations nonsensical.

References

[1] O. A. Reula and M. E. Rubio. Ill-posedness of Force-free Electrodynamics in Euler Potentials. *Physical Review D* **95**(064005):1-14, 2017.

[2] Heinz-Otto Kreiss and Jens Lorenz. Initial Boundary Value Problems and the Navier-Stokes Equations. Academic Press, San Diego, USA, ISBN 0-12-426125-6, 1989.

[3] Gilbert Strang. Necessary and insufficient conditions for well-posed Cauchy problems. *J.Differential equations*, **2** 107-114, 1966.

INVERSE PROBLEM FOR ENERGY DEPENDENT POTENTIALS

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We consider the nonlinear eigenvalue problem for the p-Laplacian operator with an energy dependent potential, which depends on the eigenvalue parameter. We estimate the asymptotic behavior of eigenvalues, and show that the set of zeros of the corresponding eigenfunctions is enough to determine the main term of the potential.

Energy cascade in turbulent flows using the sabra shell model.

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Abstract

Transfer of energy from large to small scales in turbulent flows is described as a flux of energy from small wave numbers to large wave numbers in the spectral representation of the Navier-Stokes equation

$$\partial_t u_i(\mathbf{k}) = -\imath k_j \int (\delta_{i\ell} - \frac{k_i k'_\ell}{k^2}) u_j(\mathbf{k}') u_\ell(\mathbf{k} - \mathbf{k}') \mathrm{d}\mathbf{k}' - \nu k^2 u_i(\mathbf{k}) + f_i(\mathbf{k}) \tag{1}$$

where ν is the viscosity and f_n is the external force. The problem of resolving the relevant scales in the flow corresponds in the spectral representation to determining the spectral truncation at large wave numbers. The effective number of degrees of freedom in the flow depends on the Reynolds number. The Kolmogorov scale η depends on Reynolds number as $\eta \sim Re^{-3/4}$, so the number of waves N necessary to resolve scales larger than η grows with Re as $N \sim \eta^{-3} \sim Re^{9/4}$. This means that even for moderate Reynolds numbers ~ 1000, the effective number of degrees of freedom is of the order of 10^7. A numerical simulation of the Navier Stokes equation for high Reynolds numbers is therefore impractical without some sort of reduction of the number of degrees of freedom.

Shell models of turbulence were introduced by Obukhov (1971) and Gledzer (1973). They consist of a set of ordinary differential equations structurally similar to the spectral Navier-Stokes equation (2). These models are much simpler and numerically easier to investigate than the Navier Stokes equation. For these models a scaling theory identical to the Kolmogorov theory has been developed, and they show the same kind of deviation from the Kolmogorov scaling as real turbulent systems do. Understanding the behavior of shell models in their own right might be a key for understanding the systems governed by the Navier Stokes equation. The shell models are constructed to obey the same conservation laws and symmetries as the Navier Stokes equation.