An H-based FEM-BEM formulation for a time dependent eddy current problem

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ABSTRACT

In applications related to electrical power engineering, the displacement current existing in a metallic conductor is negligible compared with the conduction current. In such situations, the displacement currents may be dropped from Maxwell's equations and one obtains a magneto-quasistatic sub-model usually called the eddy current problem.

The eddy current problem is generally posed in the whole space with decay conditions on the fields at infinity. Numerical schemes based on finite elements have to tackle with this additional difficulty. In engineering, the most common technique consists in computing the solutions in a sufficiently large box Ω (that contains the region of interest) after imposing heuristic boundary conditions on $\partial\Omega$. This strategy can lead to very large computational domains.

Recently, Hiptmair [6] and Meddahi *et al.* [7] avoided this drawback in time-harmonic eddy current problems by incorporating the far field effects through non-local boundary conditions on $\partial\Omega$. This allows one to reduce the computational domain to the conductor Ω_c where the eddy currents are to be computed. These non-local boundary conditions are based on boundary integral operators defined on $\partial\Omega_c$. In fact, both papers exploited the well-known symmetric method of Costabel [5] for the coupling of finite elements (FEM) and boundary elements (BEM). However, in [6], the eddy current problem is formulated in terms of the electric field while [7] extends the early work of Bossavit [2] and uses an $\mathbf{H}$-based model. In the two cases, the Galerkin discretization process relays on the **curl**-conforming finite element of Nédélec [8].

Even in the case of a sinusoidal supply voltage, there are occasions where it is not possible to assume a time harmonic behavior for the whole electromagnetic system. Besides, in some transient processes one may need to simulate the time variation of the electromagnetic fields and currents. These considerations lead us to undertake here the time-domain eddy current problem.

One effort of this work is to introduce and analyze a weak BEM-FEM formulation of this problem with no restrictions on conductor's topology or on the regularity of its boundary. The latter is possible by taking advantage (as in [6,7]) of the important tools provided by Buffa *et al.* in [3,4]. When the conductor Ω_c is non-simply connected, both [6] and [7] require the construction of cumbersome (and expensive) cutting surfaces in order to deal correctly with the discrete problem. Recently, Alonso *et al.* showed in [1] that the time harmonic $\mathbf{H}$-based formulation of eddy current problems (posed in a bounded domain) admits a saddle point structure that is free from the above restriction. Such a formulation is obtained by introducing a Lagrange multiplier (that inevitably increases the number of unknowns) associated to the $\mathbf{curl}$-free constrain satisfied by the magnetic field in the insulating region Ω_d surrounding the conductor. We adopt here the same point of view. We prove that this technique can be extended to the case of a time-domain eddy current problem posed in the whole space. Under mild hypotheses on the data, we exploit the well-known Theorem of J.L. Lions on the existence and uniqueness of weak solutions of linear parabolic problems to show that our BEM-FEM formulation of the time-domain eddy

current problem is well-posed. An important feature of our formulation is that the compact support of the current density $\mathbf{J}$ is not necessarily assumed to be completely contained in Ω_c or in its exterior.

This BEM-FEM formulation leads to a semi-discrete Galerkin scheme based on Nédélec's edge elements and Raviart-Thomas elements. We provide an error analysis of this semi-discrete scheme and analyze the asymptotic behavior of the error in terms of the mesh size parameter.

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