Terminal-edge algorithms: an integrated approach for mesh generation

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1 Abtract

Unstructured meshes are widely used as basic tools for solving application problems by means of either the finite element or the finite volume methods. In recent years, we have developed terminal-edge algorithms for the refinement of non-Delaunay triangulations [1, 4, 5], which generalize previous longest-edge algorithms, as well as a new family of terminal-edge algorithms for improving Delaunay triangulations, including algorithms for automatic quality triangulation, for obtuse triangulation improvement [2, 3] as needed in finite volume methods, and for approximate quality triangulation [6]. These algorithms take advantage of the properties of two related concepts: the Lepp (longest-edge propagation path) of any triangle (tetrahedron) and its associated terminal-edges. Either for improving or refining a mesh, the algorithms use a terminal-edge point selection criteria as follows. For any target element to be improved or refined, the midpoint of an associated terminal-edge is selected for point insertion. Each terminal-edge is a special edge in the mesh which is the common longest-edge of every element (triangle or tetrahedron) that shares this terminal-edge in the mesh. In the case of the refinement algorithm, mesh refinement is performed by longest-edge bisection of all the elements that share the terminal-edge, which is a very local operation. In the case of the improvement algorithms, a constrained Delaunay triangulation is assumed, and terminal-edge midpoint insertion is performed by using a constrained Delaunay algorithm. The process is repeatedly performed until the target element is destroyed in the mesh. In 2-dimensions the refinement algorithms essentially guarantee that the quality of the input triangulation is maintained, while the improvement algorithms assure that a 30 degrees triangulation can be obtained.

In research in course we work on obtaining stronger results in 2-dimensions, the generalization of the results to 3-dimensions, the parallelization of terminal-edge refinement algorithms [7], the development of a meshing tool that integrates the algorithms in 3-dimensions [8]. In this talk, the

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algorithms, their mathematical properties, some implementation issues, as well as some of their possible extensions are discussed.

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