A-priori and a-posteriori error analysis of an augmented mixed finite element method for linear elasticity*

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Abstract

We present a new stabilized mixed finite element method for the linear elasticity problem. The approach is based on the introduction of Galerkin least-squares terms arising from the constitutive and equilibrium equations, and from the relation defining the rotation in terms of the displacement. We show that the resulting augmented variational formulation and the associated Galerkin scheme are well posed, and that the latter becomes locking-free and asymptotically locking-free for Dirichlet and mixed boundary conditions, respectively. In particular, the discrete scheme allows the utilization of Raviart-Thomas spaces of lowest order for the stress tensor, piecewise linear elements for the displacement, and piecewise constants for the rotation. In the case of mixed boundary conditions, the essential one (Neumann) is imposed weakly, which yields the introduction of the trace of the displacement as a suitable Lagrange multiplier. This trace is then approximated by piecewise linear elements on an independent partition of the Neumann boundary whose mesh size needs to satisfy a compatibility condition with the mesh size associated to the triangulation of the domain. A reliable and efficient a-posteriori error estimate is also described. Finally, several numerical results illustrating the performance of the augmented scheme are reported.

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