Interpolation error estimates in $W^{1,p}$, for degenerate Q_1 isoparametric elements

Gabriel Acosta* and Gabriel Monzón

Universidad Nacional de General Sarmiento

J. M. Gutierrez 1150, Los Polvorines

Buenos Aires, Argentina.

Abstract

For convex quadrilateral elements and $1 \leq p$, the usual $W^{1,p}$ error estimate for the Q_1 isoparametric Lagrange interpolation, called hereafter Q, reads

$$\|u - Qu\|_{L^p(K)} + h|u - Qu|_{1,p,K} \leq Ch^2|u|_{2,p,K} \qquad (1)$$

where h denotes the diameter of K.

Two facts, about (1), are well known: the convexity of K is a sufficient condition to get the estimate

$$\|u - Qu\|_{L^p(K)} \leq Ch^2|u|_{2,p,K}$$

with C bounded independently on the shape of K, however

$$|u - Qu|_{1,p,K} \leq Ch|u|_{2,p,K} \qquad (2)$$

requires extra assumptions on K to keep C uniformly bounded.

In the early work by Ciarlet and Raviart [2] is proved that (2) holds for *regular* elements if the interior angles of K are bounded from 0 and π. Since then, a large number of geometrical conditions have appeared in the literature and (2) has been proved for different kinds of "degenerate" elements.

As far as we know, for the case $p = 2$, the most general condition under which (2) holds is that defined by the so called "regular decomposition property" (or shortly RDP) (see [1]).

In this talk we review most of the results concerning (2) and we show that the same conclusions obtained in [1] are valid in $W^{1,p}$ for $1 \leq p < 3$. We also give counterexamples for the case $3 \leq p$ showing that the result can not be generalized for more regular functions. Despite this fact, we prove that optimal order error estimates of the type (2) are valid for any $1 \leq p$, keeping the interior angles of the element bounded away from 0 and π, independently of the aspect ratio (i.e. without the regularity condition). We also show that this restriction on the interior angles can not be removed for $3 \leq p$.

* e-mail: gacosta@ungs.edu.ar

References

[1] Acosta, G., Durán, R. G. (2000): Error estimates for Q_1 isoparametric elements satisfying a weak angle condition, SIAM J. Numer. Anal. 38, 1073-1088.

[2] Ciarlet, P. G., Raviart, P. A. (1972): Interpolation theory over curved elements, with applications to finite elements methods, Comp. Meth. Appl. Mech. Eng.**1**, 217-249.