Multiresolution Schemes for Strongly Degenerate Parabolic Equations

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Abstract

In the present work we extend multiresolution schemes to strongly degenerate parabolic (or mixed-type parabolic-hyperbolic) equations of the type

$$u_t + \nabla \cdot \boldsymbol{f}(u) = \Delta A(u), \quad (\boldsymbol{x}, t) \in Q_T := \Omega \times [0, T], \quad \Omega \subseteq \mathbb{R}^d, \quad d = 1, 2, 3, \quad (1)$$

where $\boldsymbol{f} : \mathbb{R} \to \mathbb{R}^d$ is a piecewise smooth, Lipschitz continuous flux vector, and the diffusion function $A(\cdot)$ is assumed to be Lipschitz continuous and piecewise smooth with $A(v) \geq A(u)$ for $v > u$. We observe that $A(\cdot)$ may have flat regions, i.e., we allow that there exist intervals $[\alpha, \beta]$ with $A(u) = \text{const.}$ for all $u \in [\alpha, \beta]$, such that equation (1) degenerates into the first-order conservation law $u_t + \nabla \cdot \boldsymbol{f}(u) = 0$ on $[\alpha, \beta]$ [1]. Since degeneracy occurs on solution value intervals of positive length (and not only at isolated points), (1) is called *strongly degenerate*. Applications of (1) include mathematical models of sedimentation-consolidation processes of particulate suspensions [1], two-phase flow in porous media or traffic flow with driver reaction. Clearly, solutions of (1) are discontinuous in general, and due to its strongly degenerancy step gradients occure alog the evolution process. This behavior motivates the use of adaptive methods. We are going to considerate multiresolution schemes based on interpolating wavelet transform for point values [3] and high resolution schemes [2] based on essentially non-oscillatory (ENO) flux interpolations [4].

References

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