Complexity of Functions via their Approximation by Semialgebraic Ones

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Traditionally, complexity of functions is related to their regularity (i.e., via classical approximation theory, to the rate of their polynomial approximation).

It turns out, that many complexity properties of functions (like the geometry of their critical and near-critical points and values, average of Betti numbers of their level sets, etc.), depend in fact not on the regularity, but on the rate of their approximation by semialgebraic (and generally, by "simple") functions.

We plan to discuss some manifestations of this phenomenon, stressing those, where the difference between regularity and "complexity" becomes especially apparent: maxima of smooth families and functions on infinite-dimensional spaces.