Quadratic Forms and the Parametrization Problem

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Algebraic curves and surfaces play an important and ever increasing role in computer aided geometric design, computer vision, and computer aided manufacturing. Consequently, theoretical results need to be adapted to practical needs. We need efficient algorithms for generating, representing, manipulating, analyzing, rendering algebraic curves and surfaces. In the last years there has been dramatic progress in all areas of algebraic computation. In particular, the application of computer algebra to the design and analysis of algebraic curves and surfaces has been extremely successful.

One interesting subproblem in algebraic geometric computation is the rational parametrization of curves and surfaces. The tacnode curve defined by $f(x,y) = 2x^4 - 3x^2y + y^4 - 2y^3 + y^2$ in the real plane has the rational parametrization

$$x(t) = \frac{t^3 - 6t^2 + 9t - 2}{2t^4 - 16t^3 + 40t^2 - 32t + 9}, \quad y(t) = \frac{t^2 - 4t + 4}{2t^4 - 16t^3 + 40t^2 - 32t + 9}$$

over $\mathbb{Q}$. The criterion for parametrizability is the genus. Only curves of genus 0 have a rational parametrization, and only surfaces of arithmetic genus 0 and second plurigenus 0 have a rational parametrization.

Computing parametrizations essentially requires the full analysis of singularities (either by successive blow-ups, or by Puiseux expansion) and the determination of regular points on the curve or surface. We can control the quality of the resulting parametrization by controlling the field over which we choose these regular points. Thus, finding a regular curve point over a minimal field extension on a curve of genus 0 is one of the central problems in rational parametrization of curves, compare [SeWi97] and [HiWi98]. Similarly, finding rational curves on surfaces leads to parametrizations, compare [LSWH00] and [LSW01]. We discuss methods for finding such rational points of curves and rational curves on surfaces.

References

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