Symmetric Functions Applied to Decomposing Solution Sets of Polynomial Systems

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A fundamental problem of numerical algebraic geometry is to use floating point numerical algorithms decompose to decompose the solution set of a polynomial system, which often containing positive dimensional sets of different dimensions, into its components. Previously, the speaker with J. Verschelde and C. Wampler showed how to generate sets of generic points guaranteed to include points from every component; and how monodromy can be used to efficiently predict the partition of these points by membership in the components. However, confirmation of this prediction required an expensive procedure of sampling each component to find an interpolating polynomial that vanishes on it. Recent results of ours prove theoretically and demonstrate in practice that linear traces suffice for this verification step. This gives great improvements in both computational speed and numerical stability. Moreover, in the case that we still wish to compute an interpolating polynomial, this same work shows how to do so more efficiently by building a structured grid of samples, using divided differences, and applying symmetric functions. Several test problems illustrating the effectiveness of the new methods will also be presented.